Arithmetic sequences and series (continued)

Reminder. A sequence is an ordered list of numbers.
A series is an ordered sum of numbers (sum of the arithmetic sequence :))

1. Prove, that if $a, b, and\ c$ are in arithmetic progression, then for any constant k, the numbers $ak, bk, and\ ck$ also form an arithmetic progression.

2. An arithmetic sequence has first and last terms of 42 and -138 respectively. The common difference is 10.
   a) Find a formula for the kth term of the sequence in terms of k.
   b) Home many terms are there in the sequence?

3. Th first four terms of an arithmetic sequence, in order are $x + y, x - y, and\ x/y$.
   a) Using the first two terms, express the common difference of the sequence in terms of y.
   b) Using the common difference, find new expression for the first and fourth terms in the sequence.
   c) Using your expression for part (b), build and solve a system of equations to find x and y.
   d) Find the fifth term in the sequence.

4. Let $a, b, and\ c$ be in arithmetic progression.
   a) Prove that $2b = a + c$
   b) Prove that $a^2 - bc, b^2 - ac, and\ c^2 - ab$ are in arithmetic progression.

5. The fourth term of an arithmetic sequence is 203 and the thirteenth term is 167. What is the smallest value of n such that the nn^{th} term of the sequence is negative?

6. Show that if b is an average of a and c, then a,b, and c are in arithmetic progression.

7. Prove that is constant k is added to each term of an arithmetic progression, the resulting sequence is also arithmetic.

8. The pages of the book are numbered 1 through n. When the page numbers of the book are added. One page number was mistakenly added twice, resulting in the incorrect sum of 1986. What was the number of the page that was added twice?

9. Find the value of $a_2 + a_4 + a_6 + \cdots + a_{98}$, if $a_1, a_2, a_3, \ldots$ is an arythmetics progression with common difference 1, and $a_1 + a_2 + a_3 + \cdots + a_{98}, = 137$

10. Let n be a positive integer.

    a) Find the sum of n smallest positive odd integers
    b) Find the sum of n smallest positive odd integers greater than 100.

11. Let and b be positive integers with $a \leq b$. Find the sum of the positive integers from a to b, including a and b, in terms of a and b.

12. Let $a_1, a_2, a_3, \ldots$ and $b_1\ b_2, b_3, \ldots$ be arithmetic progressions such that $a_1 = 25, b_1 = 75, and\ a_{100} + b_{100} = 100$. Find the sum of the first 100 terms of the progression $a_1 + b_1, a_2 + b_2, \ldots$

13. For every n, the sum of the first n terms of a certain arithmetic progression is $2n + 3n^2$. Find a formula for the nth term of the progression.

14. In a certain arithmetic progression, the ratio of the sum of the first r terms to the sum of the first s terms is r^2/s^2 for any r and s. Find the ratio of the 8th term to the 23rd term.