Algebraic Expressions and Identities

Today we discussed how one works with algebraic expressions, i.e. expressions containing variables, such as $2(x+1)-3$. You probably already know most of these identities, but it is important to get them all together. In particular, we discussed the following useful formulas about exponents, radicals, and the basic algebraic identities.

1 Exponents Laws

If a is a real number, n is a positive integer, $a^n = \underbrace{a \times a \times \cdots \times a}_{n\text{-times}}$

$$a^0 = 1$$
$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(ab)^n = a^n b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$
$$a^{-n} = \frac{1}{a^n}$$
$$(a^m)^n = a^{mn}$$

2 Radicals

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}, n \neq 0$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3 Main Algebraic Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a - b)(a + b)$$

Homework problems on back

Homework

Please try to do as many of the problems below as you can, and bring completed solutions with you to next class (do not forget to put your name on it!). Some of these problems are similar to those we have discussed in class; some are new. It is OK if you can not solve some problem — but do not give up before making an effort, maybe putting the problem away and coming back to it later — which means you have to start the homework early.

Please always write solutions on a separate sheet of paper. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. The difference between two numbers is $\dfrac{5}{12}$. If $\dfrac{3}{4}$ of the larger number is $\dfrac{3}{8}$ more than $\dfrac{1}{2}$ of the smaller, find the larger number.

2. Without a calculator, compute
$$19999 \cdot 20001$$
Is there a shorter way of doing it than the straightforward multiplication?

3. Expand
   a. $2x(a + 2b + 3c)$
   b. $-3y(a - ay + by)$
   c. $(a^2 + 2a + 1)(a + 1)$
   d. $(b^2 - 2b + 1)(b - 1)$
   e. $(4x - 7y)(4x + 7y)$
   f. $(6x^2 - y)(7x^2 - 2x - 5)$

4. Factor (i.e., write as a product) the following expressions:
   a. $ac + ab$
   b. $x^2 + 3x^3$
   c. $x^2 - 2x - yx + 2y$
   d. $4x^2 - 4x + 1$
   e. $4x^2 + 16x + 2xy + 8y$
   f. $x^2(x + 4) + 5(x + 4)$
   g. $100x^8 y^2 - 16x^4 y^6$
   h. $a^2 + 4ab + 4b^2$
   i. $a^2 - 2a + 1$
   j. $x^2 - 7$ [Hint: $7 = (\sqrt{7})^2$.]
   k. $a^4 - b^4$ [Hint: $a^4 = (a^2)^2$.]

5. John takes 15 min to walk from school to the bus station. Jim takes 20 min to walk from the school to the bus station. If the difference in their speeds is 2 km/h, how far is the station from the school?

6. Simplify:
$$\text{(a)}\ \frac{1}{x+1} - \frac{1}{x-1} \qquad \text{(b)}\ \left(1 + \frac{1}{x}\right) \div (x + 1) \qquad \text{(c)}\ \left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$