Math 6d: Homework 16

Due: February 3

Please be prepared to hand in.

Just The Basics: Please make sure you are proficient with the following skills and concepts.

Logic and Proof

inference rule	tautology	name
p, $p \to q$, $\therefore\ q$	$(p \wedge (p \to q)) \to q$	Modus ponens (mode that affirms)
$\neg q$, $p \to q$, $\therefore\ \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens (mode that denies)
$p \to q$, $q \to r$, $\therefore\ p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	hypothetical syllogism
$p \vee q$, $\neg p$, $\therefore\ q$	$((p \vee q) \wedge (\neg p)) \to q$	disjunctive syllogism

p, $\therefore\ p \vee q$	$p \to (p \vee q)$	addition
$p \wedge q$, $\therefore\ p$	$(p \wedge q) \to p$	simplification
p, q, $\therefore\ p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	conjunction
$p \vee q$, $\neg p \vee r$, $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	resolution

DE MORGAN'S LAWS

NOT (A AND B) = (NOT A) OR (NOT B)

NOT (A OR B) = (NOT A) AND (NOT B)

1. In each truth table, which statement should be the heading for column 3?

   1. $p \wedge q$
   2. $p \vee q$
   3. $p \to q$
   4. $p \leftrightarrow q$

Column 1	Column 2	Column 3
p	q	?
T	T	T
T	F	F
F	T	F
F	F	T

p	q	?
T	T	T
T	F	F
F	T	F
F	F	F

Column 1	Column 2	Column 3
p	q	?
T	T	T
T	F	T
F	T	F
F	F	T

2. Which argument is *not* valid?

1. Given: $a \to b$
   a
   Conclusion: b

2. Given: $a \vee b$
   $\sim b$
   Conclusion: $\sim a$

3. Given: $a \to b$
   $\sim b$
   Conclusion: $\sim a$

4. Given: $a \to b$
   $b \to \sim c$
   Conclusion: $a \to \sim c$

Please be prepared to hand in.

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3. Fill in the logic proof below with the correct reasons.

Given: $Z \vee A$
$Z \rightarrow L$
$\sim A$
$\therefore L$

Statements	Reasons
1. $Z \vee A$	1. Given
2. $\sim A$	2. Given
3. Z	3.
4. $Z \rightarrow L$	4. Given
5. L	5.

Given: $A \rightarrow \sim (B \wedge C)$
B
$S \rightarrow C$
$P \wedge Q$
A
$\therefore \sim S \wedge Q$

Statements	Reasons
1. $A \rightarrow \sim(B \wedge C)$	1. Given
2. A	2. Given
3. $\sim (B \wedge C)$	3.
4. $\sim B \vee \sim C$	4.
5. B	5. Given
6. $\sim C$	6.
7. $S \rightarrow C$	7. Given
8. $\sim S$	8.
9. $P \wedge Q$	9. Given
10. Q	10.
11. $\sim S \wedge Q$	11.

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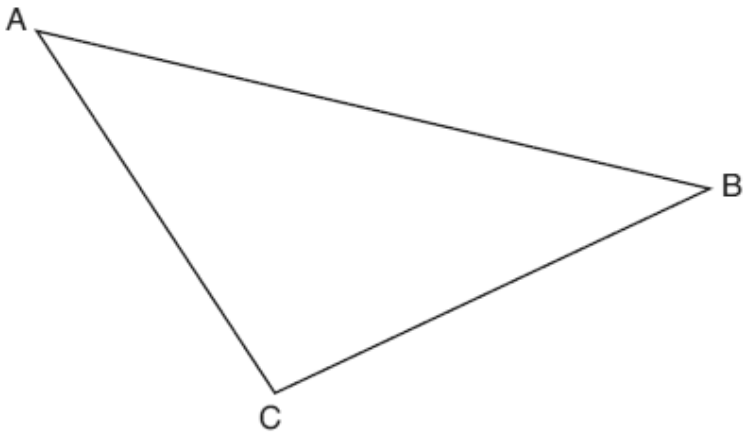
Constructions using a compass and straight-edge

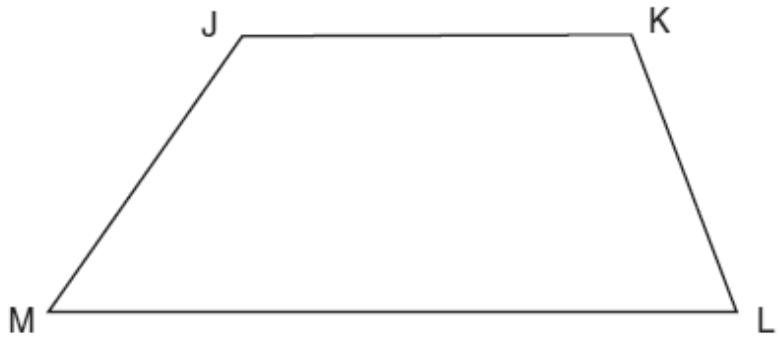
4. Using a compass and straightedge, construct the median to side $\overline{AC}$ in Δ*ABC* below.



5. Given: Trapezoid *JKLM* with $\overline{JK}$ || $\overline{ML}$.

Using a compass and straightedge, construct the altitude from vertex *J* to $\overline{ML}$.

[Leave all construction marks.]



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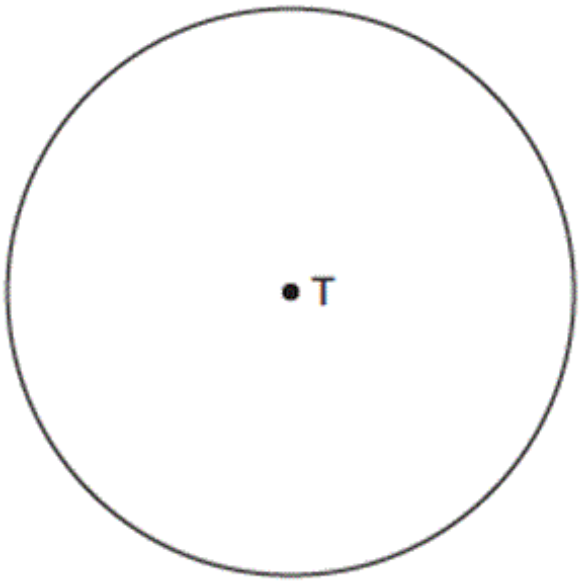
6. Construct an equilateral triangle inscribed in circle *T* shown below.

[Leave all construction marks.]



Transformations

Line Reflections:

x-axis: $(x, y) \rightarrow (x, -y)$
y-axis: $(x, y) \rightarrow (-x, y)$
the line $y = x$: $(x, y) \rightarrow (y, x)$

Rotations About the Origin:

90º counter-clockwise: $(x, y) \rightarrow (-y, x)$
180º (both clockwise and counter-clockwise): $(x, y) \rightarrow (-x, -y)$
270º counter-clockwise: $(x, y) \rightarrow (y, -x)$

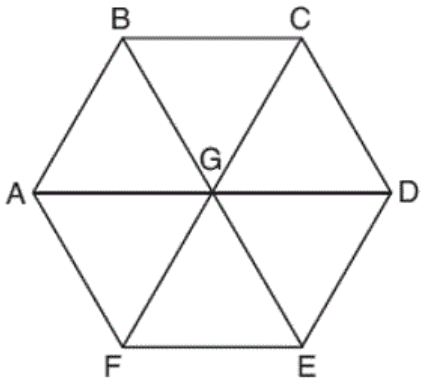
A 90º clockwise rotation is identical to a 270º counter-clockwise rotation.
A 270º clockwise rotation is identical to a 90º counter-clockwise rotation.
A 180º rotation is identical whether performed clockwise or counter-clockwise.

Translations:

A translation by a units in the horizontal direction and b units in the vertical direction: $(x, y) \rightarrow (x + a, y + b)$.

Please be prepared to hand in.

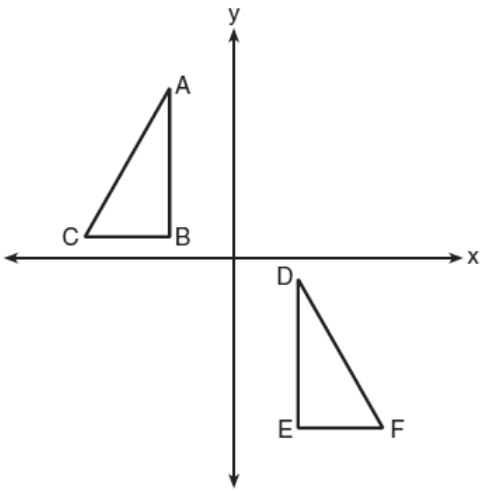
7. In regular hexagon *ABCDEF* shown below, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ all intersect at *G*.



When Δ*ABG* is reflected over $\overline{BG}$ and then rotated 180° about point *G*, Δ*ABG* is mapped onto

1. Δ*FEG*
2. Δ*AFG*
3. Δ*CBG*
4. Δ*DEG*

8. In the diagram below, Δ*ABC* ≅ Δ*DEF*.



Which sequence of transformations maps Δ*ABC* onto Δ*DEF*?

1. a reflection over the *x*-axis followed by a translation
2. a reflection over the *y*-axis followed by a translation
3. a rotation of 180° about the origin followed by a translation
4. a counterclockwise rotation of 90° about the origin followed by a translation

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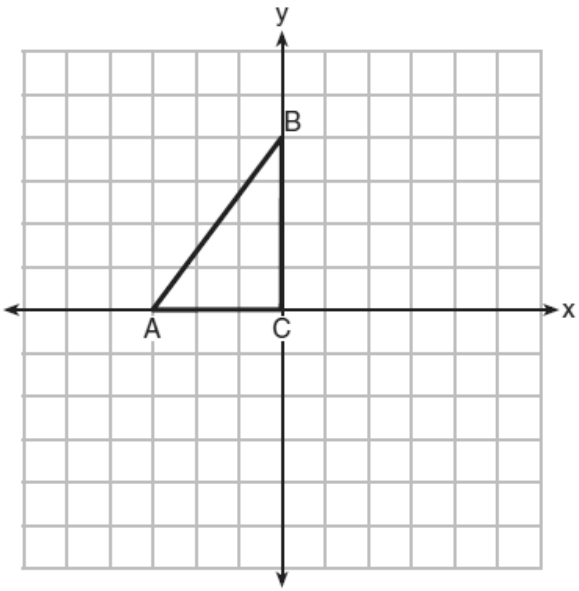
9. Triangle *ABC* is graphed on the set of axes below. Graph and label Δ*A*′*B*′*C*′, the image of Δ*ABC* after a reflection over the line *x* = 1.



Coordinate Geometry

The Distance Formula	**The Midpoint Formula**
The distance *d* between any two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	The midpoint of (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Partition a Segment

1. Label your points (x_1, y_1) and B (x_2, y_2)
   Note: since it is a directed segment, **order does matter**.
2. 2. Convert the ratio into a percent (keep as a fraction) ***a:b***
   $Percent\ ratio\ (\%) = \frac{a}{a+b}$
3. Find the rise and run for the segment (**order does matter**)
   $rise: y_2 - y_1 \quad run: x_2 - x_1$
4. To find the partitioning point:
   $x - coordinate: \boldsymbol{x_1} + \boldsymbol{run}\ (\%\ \boldsymbol{in\ fraction\ form})$
   $y - coordinate: \boldsymbol{y_1} + \boldsymbol{rise}\ (\%\ \boldsymbol{in\ fraction\ form})$

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10. Triangle *ABC* has coordinates *A*(−6,2), *B*(−3,6), and *C*(5,0). Find the perimeter of the triangle. Express your answer in simplest radical form.

1. $15 - \sqrt{125}$
2. $20\sqrt{5}$
3. $15\sqrt{125}$
4. $15 + 5\sqrt{5}$

11. Point *P* is on the directed line segment from point *X*(−6, −2) to point *Y*(6,7) and divides the segment in the ratio 1:5. What are the coordinates of point *P*?

1. $(4, 5\frac{1}{2})$
2. $(-4\frac{1}{2}, 0)$
3. $(-\frac{1}{2}, -4)$
4. $(-4, -\frac{1}{2})$

Linear Equations

Slope is equal to _____ over _____

The Slope equation is m = _____

Slope-Intercept form looks like:_____

"m" stands for the _____ and "b" stands for the _____

and the y-intercept is where my line crosses the _____

Point-Slope form looks like:_____

Standard Form looks like: _____

Parallel Lines have the _____ slope.

Perpendicular Lines have the _____ slope.

Please be prepared to hand in.

12. Line segment *NY* has endpoints *N*(−11, 5) and *Y*(5, −7). What is the equation of the perpendicular bisector of $\overline{NY}$?

1. $y + 1 = \frac{4}{3}(x + 3)$
2. $y - 6 = \frac{4}{3}(x - 8)$
3. $y + 1 = -\frac{3}{4}(x + 3)$
4. $y - 6 = -\frac{3}{4}(x - 8)$

Systems of Equations

13. Which system of equations will yield the same solution as the system below?

$x - y = 3$
$2x - 3y = -1$

1. $-2x - 2y = -6$
   $2x - 3y = -1$
2. $-2x + 2y = 3$
   $2x - 3y = -1$
3. $2x - 2y = 6$
   $2x - 3y = -1$
4. $3x + 3y = 9$
   $2x - 3y = -1$

Please be prepared to hand in.

14. At Bea's Pet Shop, the number of dogs, *d*, is initially five less than twice the number of cats, *c*. If she decides to add three more of each, the ratio of cats to dogs will be $\frac{3}{4}$.

PART A:

Write an equation or system of equations that can be used to find the number of cats and dogs Bea has in her pet shop.

PART B

Could Bea's Pet Shop initially have 15 cats and 20 dogs? Explain your reasoning.

PART C:

Determine algebraically the number of cats and the number of dogs Bea initially had in her pet shop.