Math 6d: Homework 15

Due: January 27

Please be prepared to hand in.

A system of linear equations can have a unique solution, no solution, or infinitely many solutions.

- Systems with a unique solution are comprised of two linear equations whose graphs have different slopes; that is, their graphs in a coordinate plane will be two distinct lines that intersect at only one point.

- Systems with no solutions are comprised of two linear equations whose graphs have the same slope but different y-intercept points; that is, their graphs in a coordinate plane will be two parallel lines (with no intersection).

- Systems with infinitely many solutions are comprised of two linear equations whose graphs have the same slope and the same y-intercept point; that is, their graphs in a coordinate plane are the same line (i.e., every solution to one equation will be a solution to the other equation).

Systems of linear equations can be solved by eliminating one of the variables from the system. One way to eliminate a variable is by setting both equations equal to the same variable and then writing the expressions equal to one another.

Example: Solve the system $\begin{cases} y = 3x - 4 \\ y = 2x + 1 \end{cases}$.

Since the expressions $3x - 4$ and $2x + 1$ are both equal to y, they can be set equal to each other and the new equation can be solved for x:

$$3x - 4 = 2x + 1$$

Another way to eliminate a variable is by multiplying each term of an equation by the same constant to make an equivalent equation. Then, use the equivalent equation to eliminate one of the variables and solve the system.

Example: Solve the system $\begin{cases} 2x + y = 8 \\ x + y = 10 \end{cases}$.

Multiply the second equation by -2 to eliminate the x.

$$-2(x + y = 10)$$
$$-2x - 2y = -20$$

Now we have the system $\begin{cases} 2x + y = 8 \\ -2x - 2y = -20 \end{cases}$.

When the equations are added together, the x is eliminated.

$$2x + y - 2x - 2y = 8 + (-20)$$
$$y - 2y = 8 + (-20)$$

Once a solution has been found, verify the solution graphically or by substitution.

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1. Determine the number of solutions to each system of linear equations. If the system has a solution, find it algebraically.

   a. $\begin{cases} y = \frac{3}{7}x - 8 \\ 3x - 7y = 1 \end{cases}$

   b. $\begin{cases} 6x - 7y = \frac{1}{2} \\ 12x - 14y = 1 \end{cases}$

   c. $\begin{cases} 7x - 10 = y \\ y = 5x + 12 \end{cases}$

2. One angle measures 54 more degrees than 3 times another angle. The angles are supplementary. What are their measures? (Use an algebraic solution.)

3. Some friends went to the local movie theater and bought four large buckets of popcorn and six boxes of candy. The total for the snacks was $\$46.50$. The last time you were at the theater, you bought a large bucket of popcorn and a box of candy, and the total was $\$9.75$. How much would 2 large buckets of popcorn and 3 boxes of candy cost? (Use an algebraic solution.)

4. A piece of string is 112 inches long. Isabel wants to cut it into 2 pieces so that one piece is three times as long as the other. How long is each piece? (Use an algebraic solution.)

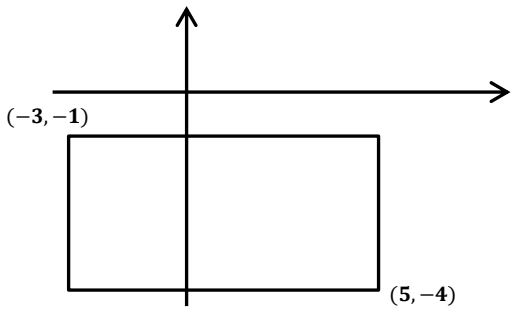
5. Consider the rectangular region:

   a. What boundary points does a line through the origin with a slope of -2 intersect? What is the length of the segment within this region along this line?

   b. What boundary points does a line through the origin with a slope of 3 intersect? What is the length of the segment within this region along this line?

   c. What boundary points does a line through the origin with a slope of $-\frac{1}{5}$ intersect?

   d. What boundary points does a line through the origin with a slope of $\frac{1}{4}$ intersect?



6. Find the point on the directed segment from $(-3, -2)$ to $(4, 8)$ that divides it into a ratio of $3:2$.

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7. Write the equation of the line through $\left(\sqrt{3}, \frac{5}{4}\right)$ and:

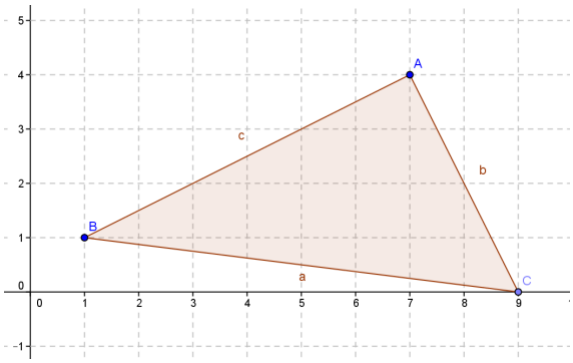
   a. Parallel to $y = 7$.

   b. Perpendicular to $y = 7$.

   c. Parallel to $\frac{1}{2}x - \frac{3}{4}y = 10$.

   d. Perpendicular to $\frac{1}{2}x - \frac{3}{4}y = 10$.

9. Given triangle ABC with vertices $(7, 4)$, $(1, 1)$, and $(9, 0)$:



   Calculate the perimeter using the distance formula.

   Calculate the area using the traditional area formula.

10. Find the perimeter of quadrilateral $DEBF$ shown below.