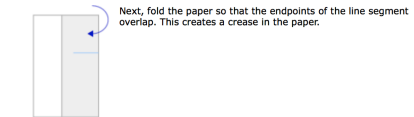
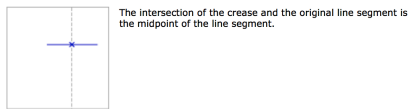


To construct the midpoint of a line segment, start by drawing a line segment on the patty paper.

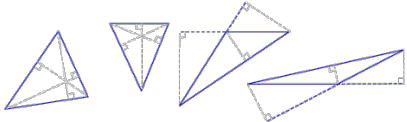


Next, fold the paper so that the endpoints of the line segment overlap. This creates a crease in the paper.

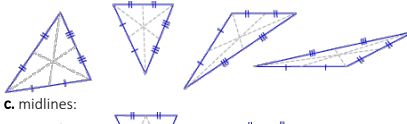


The intersection of the crease and the original line segment is the midpoint of the line segment.

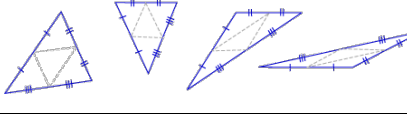
a. altitudes:



b. medians:



c. midlines:



Problem C1

Draw a line segment. Then construct a line that is

- perpendicular to it
- parallel to it
- the perpendicular bisector of the segment (A perpendicular bisector is perpendicular to the segment and bisects it; that is, it goes through the midpoint of the segment, creating two equal segments.)

Problem C2

Draw an angle on your paper. Construct its bisector. (An angle bisector is a ray that cuts the angle exactly in half, making two equal angles.)

Problem C3

Illustrate each of these definitions with a sketch using four different triangles. Try to draw four triangles that are different in significant ways – different side lengths, angles, and types of triangles. The first one in definition (a) is done as an example.

- A triangle has three altitudes, one from each vertex. (An altitude of a triangle is a line segment connecting a vertex to the line containing the opposite side and perpendicular to that side.)
- A triangle has three medians. (A median is a segment connect any vertex to the midpoint of the opposite side.)
- A triangle has three midlines. (A midline connects two consecutive midpoints.)

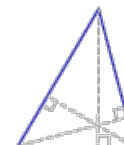
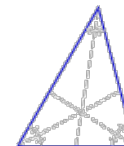
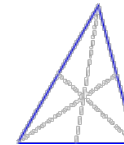
Problem C4

Draw five triangles, each on its own piece of patty paper. Use one triangle for each construction below.

- Carefully construct the three altitudes of the first triangle.
- Carefully construct the three medians of the second triangle.
- Carefully construct the three midlines of the third triangle.
- Carefully construct the three perpendicular bisectors of the fourth triangle.
- Carefully construct the three angle bisectors of the fifth triangle.

When three or more lines meet at a single point, they are said to be concurrent. The following surprising facts are true for every triangle:

- The medians are concurrent; they meet at a point called the centroid of the triangle. (This point is the center of mass for the triangle.)
- The perpendicular bisectors are concurrent; they meet at the circumcenter of the triangle. (This point is the same distance from each of the three vertices of the triangles.)
- The angle bisectors are concurrent; they meet at the incenter of the triangle. (This point is the same distance from each of the three sides of the triangles.)
- The altitudes are concurrent; they meet at the orthocenter of the triangle.



Triangles are the only figures where these concurrencies always hold. (They may hold for special polygons, but not for just any polygon of more than three sides.)

Problem C5

For each construction in parts a-d, start with a freshly drawn segment on a clean piece of patty paper. Then construct the following shapes:

- an isosceles triangle with your segment as one of the two equal sides
- an isosceles triangle whose base is your segment
- a square based on your segment
- an equilateral triangle based on your segment

Math 6d: Lesson 12



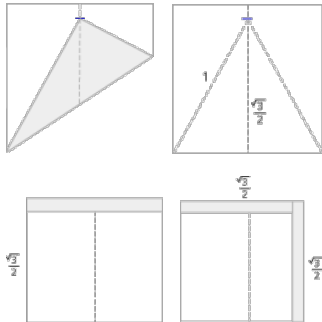
In order to obtain a square with exactly three-fourths the area of the original square (sides = 1) we need to calculate the sides of the new square:

$$a \cdot a = 3/4$$

$$a^2 = 3/4$$

$$a = \sqrt{3/4}$$

So we are looking to construct a square whose sides are equal to $\sqrt{3/4}$.



Problem C6

Start with a square sheet of paper.

- Construct a square with exactly one-fourth the area of your original square. How do you know that the new square has one-fourth the area of the original square?
- Construct a square with exactly one-half the area of your original square. How do you know that the new square has one-half the area of the original square?
- Construct a square with exactly three-fourths the area of your original square.

Problem C7

Recall that the centroid is the center of mass of a geometric figure. How could you construct the centroid of a square?

Math 6d: Lesson 12



Homework

1.	<p>Draw five quadrilaterals, each on its own piece of patty paper. Use one quadrilateral for each construction below.</p> <ol style="list-style-type: none"> Construct the eight medians of the first quadrilateral. (There will be two medians at each vertex.) Construct the four midlines of the second quadrilateral. Construct the four angle bisectors of the third quadrilateral. Construct the four perpendicular bisectors of the sides of the fourth quadrilateral. Construct four altitudes in the fifth quadrilateral -- one from each vertex.
2.	Try Problem 1 with a few different quadrilaterals. Don't just use "special" cases, like squares and rectangles. Record any observations about the constructions above. How is the situation different from what you saw with triangles?
3.	Imagine a square casting a shadow on a flat floor or wall. Can the shadow be non-square? Non-rectangular? That is, can the angles in the shadow ever vary from 90°?
4.	The math and magic of origami