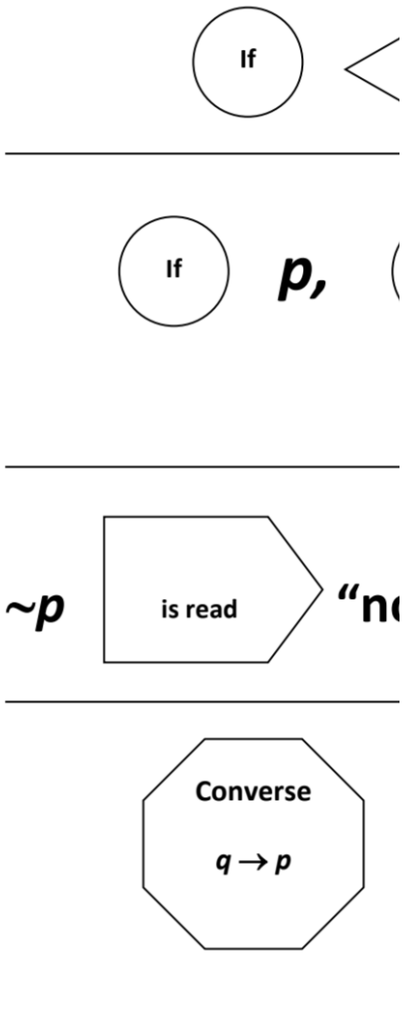
Term	Definition
Conditional Statement	A statement that can be written in "if ..., then" form.
Hypothesis	The part of a conditional statement that follows the word "if."
Conclusion	The part of a conditional statement that follows the word "then."
Negation	The opposite of a given statement formed by adding or removing the word *not* from the statement.
Negate	To add or remove the word *not* from a statement to change its truth value from true to false or from false to true.
Converse	A statement formed from a conditional statement by switching the hypothesis and the conclusion.
Inverse	A statement formed from a conditional statement by negating the hypothesis and the conclusion.
Contrapositive	A statement formed from a conditional statement by switching AND negating the hypothesis and the conclusion.
Biconditional	A statement that combines the conditional and its converse when they are both true. It uses the phrase "if and only if."

Fill in the meaning of each of the following symbols.

***p, q, r, s, t,* etc.**	Meaning: Symbols used to represent statements such as hypotheses and conclusions		
→	Meaning: if ..., then (implies)	∨	Meaning: or
~	Meaning: not	∴	Meaning: therefore
∧	Meaning: and	↔	Meaning: if and only if (IFF)

Flash Cards

Conditional Statement	***p* implies *q***
Hypothesis	**"p"**
Conclusion	**"q"**
Biconditional	***If and only if*** **Combines the conditional and its converse when both are true.**
If …, then	→
Not	~
Converse	**"Switch"** **hypothesis and conclusion**
Inverse	**"Negate"** **Hypothesis and conclusion**
Contrapositive	**"Switch and Negate"**


If
If
p,
~p
is read
“n
Converse
$q \rightarrow p$

Term	Definition	Example
Conditional Statement	An "if - then" statement	If it is Monday Then it is a school day
Hypothesis	An assumption	From above "It is Monday" is the hypothesis
Conclusion	The result of a hypothesis	From above "It is a school day" is the conclusion
Counterexample	An example that disproves a statement.	From above Thanksgiving Monday is a counterexample (no scho
Converse	A conditional statement in which the hypothesis and the conclusion are switched.	From above "If it is a school day then it is Monday"
Biconditional	A conditional statement whose converse is also true. In logic notation, a biconditional statement is written as "*p* if and only if *q*"	The statement: "If a number is even then it is divisible by 2" is true. The converse "If a number is divisible by 2, then it is even" is also true. The biconditional statement is: "A number is even if and only if it is divisible by 2"
$p \Rightarrow q$	Notation for "If p, then q" is read as "p implies q"	
$p \Leftrightarrow q$	Notation for "p if and only if q" means both the conditional statement and its converse are true.	

Term	Definition	Example
Inverse	A statement that is formed by negating both the hypothesis and the conclusion of a conditional statement.	"If a number is even, then it is divisible by 2." The inverse is: "If a number is not even, then it is not divisible by 2"
Contrapositive	A statement that is formed by negating both the hypothesis and the conclusion of the converse of a conditional statement.	"If a number is even, then it is divisible by 2." The contrapositive is: "If a number is not divisible by 2, then it is not even"
$\neg p$	"not" p	

LEARN ABOUT the Math

James and Gregory like to play soccer, regardless of the weather. Their coach made this **conditional statement** about today's practice: "If it is raining outside, then we practise indoors."

When will the coach's conditional statement be true, and when will it be false?

Example 1: Verifying a conditional statement (p.195)

Verify when the coach's conditional statement is true or false.

Hypothesis: It is raining outside

Conclusion: we practise indoors

Each of these statements is either true or false, so to verify this conditional statement, consider four cases.

Case 1: The hypothesis is true and the conclusion is true.

It rains outside and we practise indoors.

When the hypothesis and conclusion are both true, a conditional statement is true

Case 2: The hypothesis is false, and the conclusion is false.

It does not rain outside and we pratise outdoors

When the hypothesis and conclusion are both false, a conditional statement is true

Case 3: The hypothesis is false, and the conclusion is true.

It does not rain outside and we practise indoors

When the hypothesis is false and conclusion is true, a conditional statement is true

Case 4: The hypothesis is true, and the conclusion is false.

It rains outside and we practise outdoors

When the hypothesis is true and conclusion is false, a conditional statement is false

This counterexample shows that the conditional statement is false

Use a Truth Table to Summarize the Observations

Let p represent the hypothesis: *It is raining outside.*
Let q represent the conclusion: *We practise indoors.*

	p	q	$p \Rightarrow q$
	T	T	T
*	F	F	T
*	F	T	T
	T	F	F

*When the hypothesis is false, regardless of whether the conclusion is true or false, the conditional statement is **true**

From the truth table, I can see that the **only** time a **conditional statement** will be **false** is when the **hypothesis** is true and the **conclusion** is false.

Example 2: Writing conditional statements (p. 200)

"A person who cannot distinguish between certain colours is colour blind."

a) Write this sentence as a conditional statement in "if *p*, then *q*" form.

If a person cannot distinguish between certain colours then that person is colour blind.

b) Write the converse of your statement.

If a person is colour blind then that person cannot distinguish between certain colours.

c) Is your statement biconditional? Explain.

The first statement is true

The converse is true

The statement can be written:

"A person is colour blind If and only if that person cannot distinguish between certain colours."

Example 5: Verifying a biconditional statement (p. 200)

Reid stated the following biconditional statement: "A quadrilateral is a square if and only if all of its sides are equal." Is Reid's biconditional statement true? Explain.

Conditional Statement: If a quadrilateral is a square then all of its sides are equal → true

Converse: If all the sides of a quadrilateral are equal then it is a square → false

counterexample: A rhombus has 4 equal sides
∴ converse is false

Since the converse is false the biconditional statement is false.

In Summary

Key Ideas

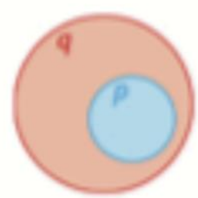
- A conditional statement consists of a hypothesis, p, and a conclusion, q. Different ways to write a conditional statement include the following:
  - If p, then q.
  - p implies q.
  - $p \Rightarrow q$
- To write the converse of a conditional statement, switch the hypothesis and the conclusion.

Need to Know

- A conditional statement is either true or false. A truth table for a conditional statement, $p \Rightarrow q$, can be set up as follows:

p	q	$p \Rightarrow q$
T	T	T
F	F	T
F	T	T
T	F	F

  A conditional statement is false only when the hypothesis is true and the conclusion is false. Otherwise, the conditional statement is true, even if the hypothesis is false.
- You can represent a conditional statement using a Venn diagram, with the inner oval representing the hypothesis and the outer oval representing the conclusion. The statement "p implies q" means that p is a subset of q.



- Only one counterexample is needed to show that a conditional statement is false.
- If a conditional statement and its converse are both true, you can combine them to create a biconditional statement using the phrase "if and only if."

1. **inverse**: A statement that is formed by negating both the hypothesis and the conclusion of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the inverse is "If a number is **not** even, then it is **not** divisible by 2."

2. **contrapositive:** A statement that is formed by negating both the hypothesis and the conclusion of the **converse** of a conditional statement; for example, for the statement "If a number is even, then it is divisible by 2," the contrapositive is "If a number is **not** divisible by 2, then it is **not** even."

p	q	$p \Rightarrow q$
T	T	T
F	F	T
F	T	T
T	F	F

Communication | *Notation*

In logic notation, the inverse of "if *p*, then *q*" is written as "If $\neg p$, then $\neg q$."

Communication | *Notation*

In logic notation, the contrapositive of "if *p*, then *q*" is written as "If $\neg q$, then $\neg p$."

Example 1: Verifying the inverse and contrapositive of a conditional Statement (p. 209)
Consider the following conditional statement: "If today is February 29, then this year is a leap year."

a) Verify the statement, or disprove it with a counterexample.

Hypothesis (p): Today is Feb. 29th

Conclusion (q): This year is a leap year

Conditional statement: if p, then q.

p	q	$p \Rightarrow q$
T	T	T

b) Verify the converse, or disprove it with a counterexample.

converse: If this year is a leap year, then today is Feb. 29th

Hypothesis (q): This year is a leap year

Conclusion (p): Today is Feb. 29th

Converse: if q, then p.

counterexample: March 12th, 2013

q	p	$q \Rightarrow p$
T	F	F

c) Verify the inverse, or disprove it with a counterexample.

Inverse: If today is not Feb. 29th, then this is not a leap year

Hypothesis ($\neg p$): Today is not Feb. 29th

Conclusion ($\neg q$): This year is not a leap year

Inverse: if $\neg p$, then $\neg q$.

counterexample: March 12th, 2013

$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	F	F

$\neg p \Rightarrow \neg q$

d) Verify the contrapositive, or disprove it with a counterexample.

Contrapositive: If this year is not a leap year, then today is not Feb. 29th

Hypothesis ($\neg q$): This year is not a leap year

Conclusion ($\neg p$): Today is not Feb. 29th

Contrapositive: if $\neg q$, then $\neg p$.

$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	T

$\neg q \Rightarrow \neg p$

memorize!

In Summary:

- You form the **inverse** of a conditional statement by negating the hypothesis and the conclusion.
- You form the **converse** of a conditional statement by exchanging the hypothesis and the conclusion.
- You form the contrapositive of a conditional statement by negating the hypothesis and the conclusion of it's converse.
- * If a conditional statement is true, then it's contrapositive is true, and vice versa.
- * If the inverse of a conditional statement is true, then the converse of the statement is also true, and vice versa.

QUIZ #2

Given: $p \rightarrow (q \vee t)$
p
$\sim q$
Prove: t

Statements	Reasons
1. $p \rightarrow (q \vee t)$	1. Given
2. p	2. Given
3. $q \vee t$	3. Law of Detachment (1,2)
4. $\sim q$	4. Given
5. t	5. Law of Disjunctive Inference (3,4)

1. Premises: $\begin{cases} \sim A \to (C \wedge D) \\ A \to B \\ \sim B \end{cases}$

Prove: C.

Statements	Reasons
1. $A \to B$	1. Given
2. $\sim B$	2. Given
3. $\sim A$	3. Law of Modus Tollens (1,2)
4. $\sim A \to (C \wedge D)$	4. Given
5. $C \wedge D$	5. Law of Detachment (4,3)
6. C	6. Law of Simplification (5)

Example. Premises: $\begin{cases} P \wedge Q \\ P \to \sim(Q \wedge R) \\ S \to R \end{cases}$

Prove: $\sim S$.

Statements	Reason
1. $P \wedge Q$	1. Given
2. P	2. Law of Simplification (1)
3. $P \to \sim(Q \wedge R)$	3. Given
4. $\sim(Q \wedge R)$	4. Law of Detachment (3,2)
5. $\sim Q \vee \sim R$	5. De Morgan's Law (4)
6. Q	6. Simplification (1)
7. $\sim R$	7. Law of Disjunctive Inference (5,6
8. $S \longrightarrow R$	8. Given
9. $\sim S$	9. Law of Modus Tollens (8,7)

3. **Example.** Premises: $\begin{cases} \sim(A \vee B) \to C \\ \sim A \\ \sim C \end{cases}$

Prove: B.

Statements	Reasons
1. $\sim(A \vee B) \to C$	1. Given
2. $\sim C$	2. Given
3. $A \vee B$	3. Law of Modus Tollens (1,2)
4. $\sim A$	4. Given
5. B	5. Law of Disjunctive Inference (3,4

5. Prove $[(t \to w) \wedge \sim w] \to \sim t$ by setting up a truth table.

t	w	~t	~w	$t \to w$	$(t \to w) \wedge \sim w$	$(t \to w) \wedge \sim w \to \sim t$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

6. Set up a truth table to prove the chain rule is valid. **(LEVEL C – Problem)**

$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$							
p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	See above
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T