Math 5c: Classwork 24
Homework #24 is due May 5.

Probability theory continued:

Addition Rule:

1. The probability of two mutually exclusive events is denoted by:

P (Y or Z) = P(Y) + P(Z)

2. The probability of two non-mutually exclusive events is denoted by:

P(Y or Z) =P(Y) + P(Z) −P (Y and Z)

Product Rule:

The probability of two independent events is denoted by:

P (Y and Z) = P(Y) * P(Z)

Question: we roll two dice. What is the probability of rolling a 5 and a 6?
Answer: There are two ways of getting a 5 and a 6: as pair (5, 6) (5 on die number 1, 6 on die number 2) or as (6, 5) (6 on die number 1, 5 on die number 2). Thus, the answer is $\frac{2}{36} = \frac{1}{18}$.

Question: we roll two dice. What is the probability of getting sum of two numbers equal to 4?
Answer: there 3 ways of getting sum 4: (1, 3), (2, 2), (3, 1). Thus, the probability is $\frac{3}{36} = \frac{1}{12}$.

Question: if one tosses a coin 10 times, what is the probability that all will be heads?
Answer: $\frac{1}{2^{10}}$ (using a calculator, one can compute that this is 1/1024 ≈ 0.001, or 1/10 of 1%).

Question: if one tosses a coin 10 times, what is the probability that all will be tails?
Answer: The same as all heads.

Question: if one tosses a coin 10 times, what is the probability that *at least* one will be heads?
Answer: Unfortunately, there are very many combinations which give at least one heads. In fact, it is easier to say which combinations do not give at least one heads: there is exactly one such combination, all tails which has a probability of $1\frac{1}{2^{10}} = \frac{1}{1024}$. The remaining combinations will give at least one heads; thus probability of obtaining at least one head is $1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.999$

Percentages and Fractions

So far we have mostly expressed probabilities as fractions. They can also be written as decimal numbers (between 0 and 1): for example, $\frac{1}{5} = \frac{2}{10} = 0.2$.

It is also common to express probabilities as percentages: by definition, 1% = $\frac{1}{100} = 0.01$ so $x\% = \frac{x}{100}$. For example, $3\% = \frac{3}{100} = 0.03$, and $1.5\% = \frac{1.5}{100} = 0.015$. This conversion is necessary when you multiply probabilities as the following example shows:

> Question. The probability of winning in a certain game is p = 5%. What is the probability of winning two times in a row?
> Answer. According to multiplication rule it is $p \times p = p^2$. However, the answer $5\% \times 5\% = 25\%$ is wrong. Correct answer is $\frac{5}{100} \times \frac{5}{100} = 0.0025$.

To convert from decimals to percent, multiply by 100: $p = (p \times 100)\%$. For example, $\frac{1}{5} = 0.2 = (0.2 \times 100)\% = 20\%$

Choosing with repetitions

Problem: how many 3-letter combinations can be formed using 26 letters of Latin alphabet?
Answer: there are 26 possibilities for the first letter, 26 for the second, and so on — so according to the product rule, there are $(26)^3$ possible combinations.

Homework

1. Simplify the fractions, using factorization rules:
   (a) $\frac{169-y^4}{y^4-12y^2-13} =$
   (b) $\frac{x^2-2x+1}{x^2-1} =$
   (c) $\frac{x^3-x}{x^2+x} =$

2. If we roll two dice, what is the probability that the product of two numbers is a multiple of 3? a multiple of 5?

3. Recall that a roulette wheel has 37 slots: 0 through 36. Among slots 1–36, half are red, the other half black (zero has no color). What is the probability of obtaining
   (a) red (on a single run of roulette)
   (b) red, then black, then 0 (on 3 successive runs)
   (c) red 15 times in a row?
   (d) blue (on a single run of roulette)
   (e) this sequence of colors: RRRBRBRBBRBBRBR (also of length 15)?

4. A hunter is shooting ducks. The probability of hitting a duck with one shot is $p = \frac{1}{3}$.
   (a) What is the probability of missing the duck (with one shot)?
   (b) He makes 5 shots. What is the probability that he misses all five times?
   (c) What is the probability that out of 5 shots, he will hit at least once? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
   (d) What is the probability that out of 5 shots, he will hit exactly once? Will this probability double if he makes 10 shots?
   (e) What is the probability that out of 5 shots, he will hit *at least* twice? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)
   (f) What is the probability that out of 5 shots, he will hit exactly twice? Will this probability double if he makes 10 shots? (You can use the calculator for computing the answers)

5. A license plate consists of 3 letters, followed by three digits. How many possible license plates are there?

6. In one kind of lottery, they put balls with numbers 1 through 100 in a bag and then draw six balls at random (the drawn ball is put aside and not returned to the bag). To win the lottery, one needs to guess all six numbers in correct order. What is the probability of this?

7. * Assume we have a thousand quarters lying on the table, 990 heads and 10 tails (facing us, the viewers). A blindfolded person cannot see whether the quarters are facing us with a head or a tail, but he/she has special gloves that allow him / her to verify that a quarter is lying on the table flat (not on the side, etc). What does the blindfolded person need to do to split given quarters into two groups with the **equal** number of tails in each group?