Math 5C: Classwork 18
Homework #18 is due March 3-rd

Square-roots

The square-root of 2 ($\sqrt{2}$) is not a rational number, i.e. it cannot be written as a fraction. Let us assume that $\sqrt{2} = \frac{p}{q}$ where p and q are some whole numbers and the fraction $\frac{p}{q}$ cannot be simplified further. We can write:

$$\left(\sqrt{2}\right)^2 = \left(\frac{p}{q}\right)^2$$

$$2 = \frac{p^2}{q^2}$$

So that: $$2q^2 = p^2$$

Thus, p must be an even number and could be rewritten as: $p = 2m$. Substituting:

$$2q^2 = p^2 = 4m^2$$

So that: $$q^2 = 2m^2$$

Thus, q must be an even number. This *contradicts* our initial assertion that $\frac{p}{q}$ could not be simplified further (at least each p, q could be reduced by one factor of 2 each). Therefore, we have proven by contradiction that $\sqrt{2}$ cannot be written as a rational number.

Rational numbers

Definition: A **rational** number is a number that can be in the form p/q where p and q are integers and q is not equal to zero.

Example: 2/3 is a rational number because 3 and 2 are both integers

Pigeonhole principle states that if ***n*** items are put **into m** pigeonholes with ***n* > *m***, then at least one pigeonhole must contain more than one item.

Theorem: any rational number is a finite or repeating decimal. The way we proved is using **Pigeonhole principle**.

Review

1. Operations with powers:

$$a^n = a \cdot a \cdot \cdots \cdot a \; (n times)$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$a^m \cdot a^n = a^{m+n};$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

2. We reviewed solving equations and solving rational equations by multiplying both sides of the equation with the denominator, for example.

$$\frac{(x+1)}{3} = 7$$

$$\frac{(x+1)}{3} \times 3 = 7 \times 3$$

$$(x+1) = 21$$

$$x = 20$$

We also revised the ***identities***:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

And ***factorizing***:

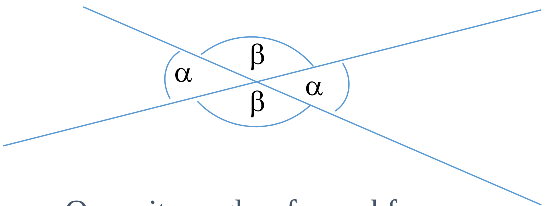
$$a(b+c) = ab + ac$$

… and used them to solve equations.

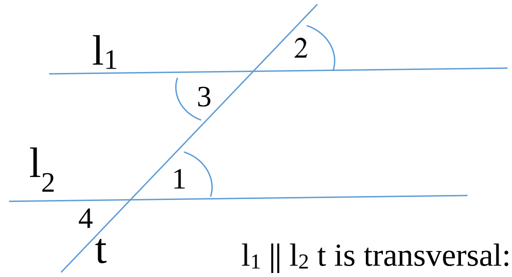
We solved equations with exponents: $a^x = a^c$ and found out that if we have equal bases we need only compare the exponents (powers) to find the unknown: $x = c$.

So, we need to find a way to rewrite the equations where both sides have the same base.

Geometry: Angles



Opposite angles, formed from crossing straight lines, are equal.



$l_1 \parallel l_2$ t is transversal:
$\angle 1 = \angle 2 = \angle 3$

$\angle\alpha = \angle\alpha$ – opposite

$\angle\alpha + \angle\beta = 180^0$ – on a straight line,

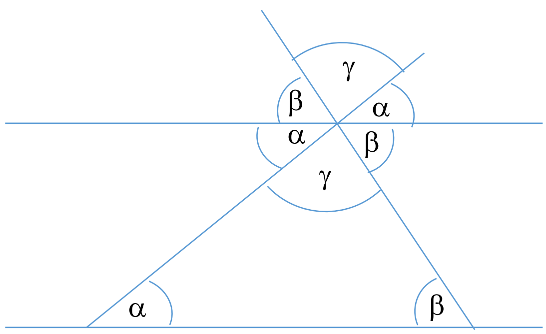
Or complementary angles

$\angle 1 = \angle 3$ = alternate internal angles

$\angle 1 = \angle 2$ = corresponding angles

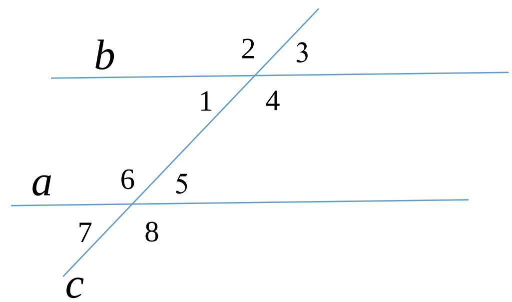
$\angle 4 = \angle 2$ = alternate exterior angles

From both these pieces of information we can show that the sum of angles in a triangle is always 180^o.



Homework

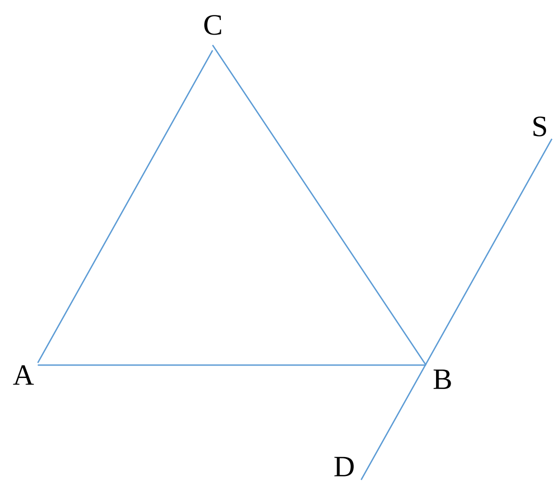
1. On the picture, *a* and *b*, which are parallel to each other, are intersected by line *c*. What are the relationships:
   (a) $\angle 3$ and $\angle 5$
   (b) $\angle 2$ and $\angle 8$
   (c) Prove that $\angle 4 + \angle 5 = 180^{o}$.



2. In the same picture,
   (a) if $\angle 7 = 65^{o}$, find: $\angle 1$, $\angle 3$, $\angle 1 + \angle 6$
   (b) If you know that $\angle 7 = \angle 1$, prove that*: $\angle 1 = \angle 3$ and $\angle 5 = \angle 1$

   (* *or say why the angles will be equal*)

3. Intersecting at point B on triangle ABC is drawn line DS, such that DS is parallel to AC. Prove that (or say why the angles will be equal):
   (a) $\angle ACB = \angle SBC$
   (b) $\angle CAB = \angle DBA$
   (c) $\angle CAB = \angle SBK$
   (d) If $\angle CAB = 40^{o}$ and $\angle BCA = 60^{o}$, find angles $\angle ABD$ and $\angle SBC$



4. In triangle ABC, $\angle A = 35^{o}$, $\angle B = 55^{o}$, prove that this triangle is right-angled.

5. What type of triangle has one angle equal to the sum of the other two?

6. Find each of the outside angles of a right-triangle, if one of its angles is 58^{o}.

7. Consider the sequence 7, 72, 73, . . .7n . . .

   (a) Show that there will be two numbers in this sequence which have the same last two digits. [*Hint: pigeonhole principle!*]

   (b) Show that from some moment, the last two digits of numbers in this sequence will start repeating periodically.