

Math 5c: Classwork 17
Homework #17 is due February 24

Definition: A **rational** number is a number that can be in the form p/q where p and q are integers and q is not equal to zero.

Example: $2/3$ is a rational number because 3 and 2 are both integers

Review

1. Operations with powers:

$$a^n = a \cdot a \cdot \dots \cdot a \text{ (n times)}$$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$a^m \cdot a^n = a^{m+n};$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

2. We reviewed solving equations and solving rational equations by multiplying both sides of the equation with the denominator, for example.

$$\frac{(x+1)}{3} = 7$$

$$\frac{(x+1)}{3} \times 3 = 7 \times 3$$

$$(x+1) = 21$$

$$x = 20$$

We also revised the **identities**:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

And **factorizing**:

$$a(b+c) = ab + ac$$

... and used them to solve equations.

We solved equations with exponents: $a^x = a^c$ and found out that if we have equal bases we need only compare the exponents (powers) to find the unknown: $x = c$.

So, we need to find a way to rewrite the equations where both sides have the same base.

Homework

1. Can one cut from the circle of diameter 9.9 m?

- a) A rectangle with the sides of 6m and 8m;
- b) A square with the side of 7m;

Please provide the step by step solution similar to how we did it in class.

2. The area of an isosceles (2 equal sides -legs) triangle is 60 square meters. The height drawn to the base (third side) is 5m. *The height of an isosceles triangle divides its base side in half. Find all sides of the triangle. Please write out the full solution similar to how it was done in class.*

The area of the triangle is **half of the base times height**.

- "b" is the distance along the base
- "h" is the height (measured at right angles to the base)

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

3. Solve the following equations for *(don't forget to check your solutions)*:

(a) $\frac{12-10x}{1-5x} = 4$

(b) $\frac{7x-9}{1-3x} - 8 = 4$

(c) $\frac{6y-18}{10y-30} - y = 2$

(d) $\frac{35z-14}{4-10z} + 3z = 2$

(e) $\frac{5z+b}{2b-3z} - 3 = -1$

4. Solve the equation (*after reading review section above*):

$$(x + 7)^2 - (x - 6)(x + 6) = 1$$

5. Simplify the fractions using the above identities and factoring rules (*after reading review section above*):

$$(a) \frac{4y^2 - 25}{6y + 15} =$$

$$(b) \frac{9a^2 + 24a + 16}{32 - 18a^2} =$$

$$(c) \frac{56z^2 - 105z}{225 - 64z^2} =$$

$$(d) \frac{36 - 60x + 25x^2}{15x - 18} =$$

6. Find n for

$$(a) 5^{-2n+3} = 125$$

$$(b) 5^{-8n} = 1/625$$

$$(c) 8^{-5n-7} = 64^9$$

7. Find the following square-roots. For the final answer if you cannot calculate the exact value of the square root, leave it as a square root.

$$(a) \sqrt{11^{28}}$$

$$(b) (\sqrt{11})^{28}$$

$$(c) \sqrt{11^{28}} / \sqrt{11^{84}}$$

$$(d) \frac{\sqrt{11^{20}}}{121}$$

$$(e) \frac{\sqrt{11^{20}}}{121} * \sqrt{11^{15}}$$

$$(f) \frac{\sqrt{11^{20}}}{121} * \sqrt{11 * 11^{15}}$$

$$(g) \frac{\sqrt{11^{20}}}{121} * \sqrt{11 * 11^{-15}}$$

$$(h) \frac{\sqrt{11^{-20}}}{121} * \sqrt{11 * 11^{-17}} / \sqrt{11^{-15-22} / 11^9}$$

8. Find the following square-roots. For the final answer if you cannot calculate the exact value of the square root, leave it as a square root.

(a) $2 / (x + 2) - 2 / (x - 2)$

(b) $(2 + 2 / x) / (x + 1)$

(c) $(2 + 2 / x) / (x + 2)$

(d) $(2 + 2 / x) / (2 - 2 / x)$

9. If $a = 7^{-32}5^{19}$, $b = 21^{35}3^{-43}$, $c = 35^{72}2^{-23}$, and $d = 10^{19}2^{44}3^{-76}$ what is the value of ab ? of a/b ? abc ? ab/c ? $abcd$? ab/cd ? Prime factors only, simplify as much as possible.

10. * Sri put a thousand quarters on a table – 990 tails and 10 heads. He's blindfolded and cannot see what's on the table, but has special gloves that allow him to identify a quarter. What does Sri need to do in order to be 100% certain that if he splits the quarters into 2 piles, each pile would have an equal number of heads.