Math 5c: Classwork & Homework 12
Homework #12 is due January 13.

Review

Powers: $a^n = a \times a \times a \times ... \times a$ (n times)

$a^0 = 1$ read: a-to-the-zero
$a^1 = a$ is just itself 'a'
$(ab)^n = a^n \times b^n$
$a^n a^m = a^{n+m}$
$\dfrac{a^n}{a^m} = a^{n-m}$
$a^n = \dfrac{1}{a^{-n}}$, $a^{-n} = \dfrac{1}{a^n}$

1. If $a = 2^{19}3^{-91}$ and $b = 2^{-46}3^{-68}$ what is the value of ab? of a/b?

Difference of squares formula: $(x - a)(x + a) = (x^2 - a^2)$
Square of the difference formula: $(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$
Square of the sum formula: $(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$

Binary Numbers: Numbers represented by using only 0s and 1s.
Powers of 2

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2^n	*1*	*2*	*4*	*8*	*16*	*32*	*64*	*128*	*256*	*512*

Example: Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation): $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$
Similarly, to convert a number into a binary, we need to represent it in powers of 2:
  $351 = \mathbf{256} + 95 = \mathbf{256 + 64} + 31 = \mathbf{256 + 64 + 16} + 15 = \mathbf{256 + 64 + 16 + 8} + 7 = \mathbf{256 +}$
  $\mathbf{64 + 16 + 8 + 4 + 2 + 1}$
  $351 = 1{\times}2^8 + 0{\times}2^7 + 1{\times}2^6 + 0{\times}2^5 + 1{\times}2^4 + 1{\times}2^3 + 1{\times}2^2 + 1{\times}2^1 + 1{\times}2^0 = 101011111$

To convert number from binary to decimal we use the familiar rule where we multiply each digit by the position value in base 2. For example:
  $1010 = 1{\times}2^3 + 0{\times}2^2 + 1{\times}2^1 + 0{\times}2^0 = 8 + 0 + 2 + 0 = 10$

Classwork

1. Simplify the following and show the answer in the exponent (power) form

(a) $\dfrac{3^7 \cdot 2^7}{2^3 \cdot 2^4} =$

(b) $\dfrac{6^5 \cdot 2^4}{3^5 \cdot 2^2} =$

(c) $\dfrac{7^9 \cdot 2^5}{7^2 \cdot 2^4} =$

(d) $\dfrac{11^4}{11^2 \cdot 5^2 \cdot 5^3} =$

2. Solve the equations:

(a) $\dfrac{9}{13} z = 3$

(b) $|x| = 2$

(c) $|x + 11| = 2$

(d) $\dfrac{x}{2} + 1 = \dfrac{4x}{7}$

(e) $x = \dfrac{1}{4} x + 6$

(f) $\dfrac{x+3}{x+1} = 4$

3. Convert the decimal numbers to binary:

   9, 12

4. Convert the following binary numbers to decimal:

   101, 1001

5. Base 4 numbers:

   a) add two base 4 numbers together:

$$\begin{array}{r} 123 \\ +\ 321 \\ \hline \end{array} \qquad \begin{array}{r} 3201 \\ +\ 2310 \\ \hline \end{array}$$

   *[Do not add in base 10 and translate the result to base 4, try performing addition in base 4, **think base 4**]*

Homework 12 problems

1. Solve the following equations:

   a) $3(x - 1) - 4 = 3x + 8$

   b) $5(x - 1) - 4 = 3x + 8$

c) $\frac{1}{2}(x-1) = -19$

d) $|2x| = 10$

2. Simplify the following and show the answer in the exponent (power) form

(a) $7^4 \cdot 11^2 \cdot 11^{-5} \cdot 7^2 =$

(b) $\dfrac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} =$

(c) $\dfrac{42^2}{6^2} =$

(d) $\dfrac{3^5 \cdot 3^{-5}}{3^9} =$

(e) $\dfrac{x^2 \cdot y^2 \cdot x^{-3}}{x^2} =$

3. Convert the decimal numbers to binary:

$9 =$

$12 =$

$24 =$

$38 =$

$45 =$

4. Convert the following binary numbers to decimal:

$101 =$

$1001 =$

$10110 =$

11011 =

10101 =

5. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number.]

$99^2 - 9^2 =$

6. Can you find whole numbers a; b such that $a^2 - b^2 = 17$? [Hint: use the formula we talked about in class, and think what $a - b$ and $a + b$ must be.]

$a^2 - b^2 = 17$ a =   b =

7. For the following problem, you need to know that the speed of light is about 300,000 km/sec, and one year is about 3×10^7 seconds.
   a) In astronomy, a common unit of distance is a light year: the distance light covers in one year. How many kilometers is it?

   b) Another common unit of distance in astronomy is a parsec, which is approximately equal to 3×10^{13} km.

- Can you compute how many parsecs are there in one light year?

- How many parsecs is the distance between Earth and Sun (The distance is about 1.5×10^8km)?

- How many parsecs is the distance between Earth and the Andromeda Nebula (2,000,000,000,000,000,000,000 km)?

8. If $a = 3^{19}5^{-91}$, $b = 2^{-46}3^{-68}$, $c = 2^{54}5^{-8}$, and $d = 10^7 2^{-4}3^{-5}$ what is the value of ab? of a/b? abc? ab/c? abcd? ab/cd?

$ab =$

$a/b =$

abc =

ab/c =

abcd =

ab/cd =

9. How many zeroes does the number $4^{15}5^{26}$ end with?

10. Base 4 numbers:
    a) add two base 4 numbers together:
    *[Do not add in base 10 and translate the result to base 4, try performing
    addition in base 4, **think base 4**]*

$$
\begin{array}{r}
123 \\
+\ 331 \\
\hline
\end{array}
\qquad\qquad\qquad
\begin{array}{r}
3201 \\
+\ 2313 \\
\hline
\end{array}
$$

    b) Write a formula, instruction, or algorithm on how to translate base 4 number
       abcd to base 10 number, where a, b, c, d can be 0, 1, 2, or 3.

c) Translate the numbers and the results from a) into the base-10 system

$123_4 =$

$331_4 =$

$result1 =$

$3201_4 =$

$2313_4 =$

$result2 =$