

Example of substitution when solving equations:

Example: $(y + 5) \div 3 = 7$

substitution: $y + 5 = z$

$$z \div 3 = 7$$

$$z = 7 \times 3 = 21$$

$$y + 5 = 21$$

$$y = 21 - 5 = 16 \quad \text{Check: } (16 + 5) \div 3 = 7$$

Algebra. Fractions.

A fraction (from Latin: fractus, "broken") is a part of a whole.

Slice a pizza, and we get fractions:

 $\frac{1}{2}$

(One-Half)

 $\frac{1}{4}$

(One-Quarter)

 $\frac{3}{8}$

(Three-Eighths)

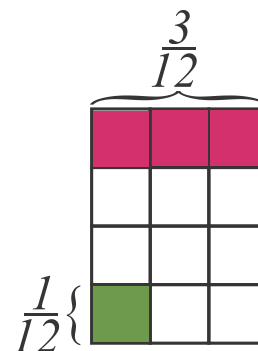
The top number says how many slices we **have**.

The bottom number says how many equal slices the whole pizza was **cut into**

Look at the picture on the right:

A chocolate bar is divided into 12 equal pieces

Each piece then would be:



$$1 \text{ (whole chocolate bar)} \div 12 \text{ (equal parts)}$$

$$= \frac{1 \text{ (whole chocolate bar)}}{12 \text{ (equal parts)}}$$

$$= \frac{1}{12} \text{ (of whole chocolate bar)}$$

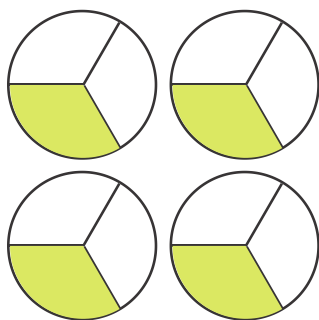
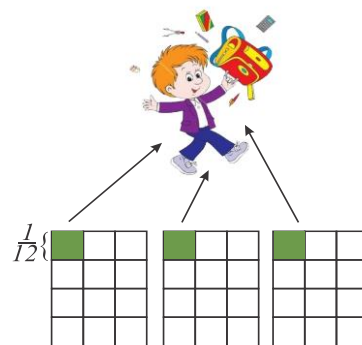
$$\text{Three pieces would be } \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = 3 \times \frac{1}{12} = \frac{3}{12} = \frac{1}{4} = 3 \div 12$$

And the whole chocolate bar would be $\frac{12}{12}=1$. It is like dividing $12:12=1$

To divide 3 chocolate bars between 12 kids we can give

each kid $\frac{1}{12}$ of each chocolate bar and that will add up to $\frac{3}{12}$

$$3 \div 12 = 3 \times \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$



To divide 4 pizzas equally between 3 friends we will give each friend $\frac{1}{3}$ of each pizza.

Each friend will get $4 \div 3 = 4 \times \frac{1}{3} = \frac{4}{3}$ which is exactly 1 whole pizza ($3 \times \frac{1}{3} = \frac{3}{3} = 1$) and $\frac{1}{3}$.

Sometimes we want to find a part of something which is not 1, but can be considered as a single object. For example, $\frac{2}{5}$ of my 30 pencils are yellow. How many yellow pencils do I have? What does it mean to find $\frac{2}{5}$ out of 30? All 30 of these pencils is a single object. We want to calculate how many pencils does a little pile of $\frac{2}{5}$ of 30 contain?



1. First, we find how many pencils will be in $\frac{1}{5}$

$$30 \times \frac{1}{5} = \frac{30}{5} = 30 \div 5$$

2. Then, we find $\frac{2}{5}$ of 30 pencils, which will be twice more

$$30 \times \frac{1}{5} \times 2 = 30 \times \frac{2}{5} = 30 \div 5 \times 2$$

Equivalent fractions



What part of the first bar is blue? $\frac{1}{3}$



What part of the second bar is blue? $\frac{4}{12}$

$\frac{1}{3} = \frac{4}{12}$ **Equivalent fractions.** Did you notice something?

Well, if you multiply both the numerator and denominator of $\frac{1}{3}$ by 4

$$\frac{1 \times 4}{3 \times 4} = \frac{4}{12} \quad \text{since } \frac{4}{4} = 1 \text{ our fraction is the same}$$

If we multiply both the numerator and denominator of a fraction by the same non-zero number (it would be the same as multiplying the fraction by 1), the fraction will not change.

Simplification of fractions

$$\frac{4}{12} \div 1 = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$$

If we divide numerator and denominator of a fraction by the same number (if they have a **common factor**), the fraction will not change. This process is called **simplification of fractions**.

Simplify:

$$\frac{27}{108} = \frac{27 \div 3}{108 \div 3} = \frac{9 \div 3}{36 \div 3} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

What are we doing? Could this be done faster, let's say in one step?

Remember what GCF is? Greatest Common Factor! Find the GCF for 27 and 108!

Now, we divide numerator and denominator of $\frac{27}{108}$ by.....