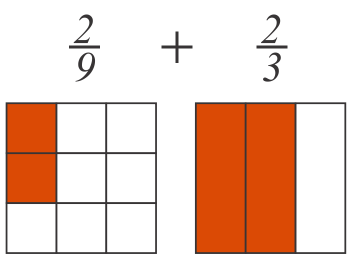
Handout # 8. **November 11, 2018**

Addition and subtraction of fractions with unlike denominators.

Can we easily compute $\frac{2}{9}+\frac{3}{9}$? Let's now try to add $\frac{2}{9}$ and $\frac{2}{3}$.



Are two fractions $\frac{2}{9}$ and $\frac{2}{3}$ similar objects?

$$\frac{2}{9}=\frac{1}{9}+\frac{1}{9} \qquad \frac{2}{3}=\frac{1}{3}+\frac{1}{3}$$

How we can calculate?

$$\frac{2}{9}+\frac{2}{3}=\frac{1}{9}+\frac{1}{9}+\frac{1}{3}+\frac{1}{3}$$

- To be able to add two fractions we must rewrite them as fractions with **the same denominator**.
- The best choice for such common denominator is the **least common multiple (LCM)** of the denominators of original fractions.
- When you find the factor by which you need to multiply your denominator, remember to multiply the numerator by the same number so that your fraction remains the same
- For example,

$$\frac{3}{9}+\frac{2}{3}=\frac{3}{9}+\frac{2\text{x}3}{3\text{x}3}==\frac{3}{9}+\frac{6}{9}=1$$

Multiplication of a whole number by a fraction.

$$\frac{2}{3}\times 5=\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}\ (we\ add\ \frac{2}{3}\ to\ itself\ 5\ times)$$

Of course we remember how to add fractions with the same denominator:

$$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{2+2+2+2+2}{3}$$

$$\frac{2+2+2+2+2}{3}=\frac{2\times 5}{3}$$

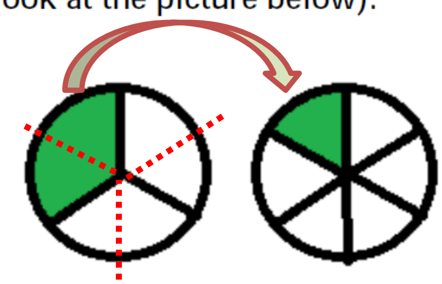
$$\boldsymbol{\frac{2}{3}\times 5=\frac{2\times 5}{3}}$$

To multiply fraction by a whole number, multiply the numerator by this number

$$\frac{a}{b}\times c=\frac{a}{b}\times\frac{c}{1}=\frac{a\ \mathrm{x}\ c}{b}$$

Multiplication of a fraction by a fraction.

Analogously, $\frac{1}{2}\times\frac{1}{3}$ means $\frac{1}{2}\,of\,\frac{1}{3}$. Now, half of 1/3 piece of a disk is 1/6 of a disk (look at the picture below).



Notice that we could have just multiplied the denominators of $\frac{1}{2}$ and $\frac{1}{3}$.

To multiply fraction by a fraction, multiply the numerators to get the numerator for the answer, multiply denominators to get denominator for the answer.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \text{ x } c}{b \text{ x } d}$$

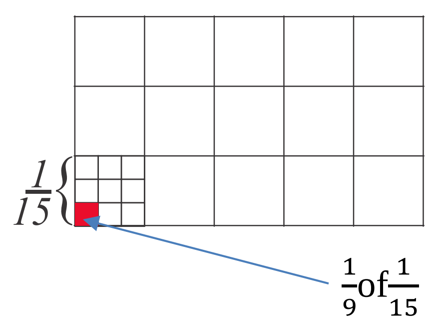
Reciprocal Fractions:

Two fractions are called reciprocal if their product is equal to 1.

$$\frac{3}{5} \times \frac{5}{3} = 1, \qquad \frac{a}{b} \times \frac{b}{a} = 1$$

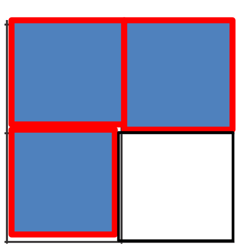
If I multiply two fractions (let say $\frac{1}{15}$ by $\frac{1}{9}$), I will find one ninth part of one fifteenth (or one fifteenth part of one ninth).

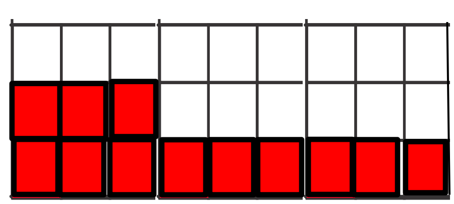
$$\frac{1}{15} \cdot \frac{1}{9} = \frac{1}{15} \div 9 = \frac{1}{15 \cdot 9} = \frac{1}{135}$$



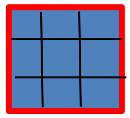
To divide one fraction by another (if they have common denominator) you need to divide only their numerators

$$\frac{3}{4} : \frac{1}{4} = 3 : 1 = 3 \qquad\qquad \frac{12}{27} : \frac{6}{27} = 12 : 6 = 2$$

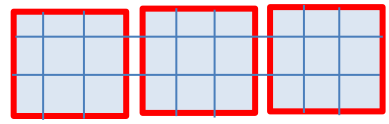




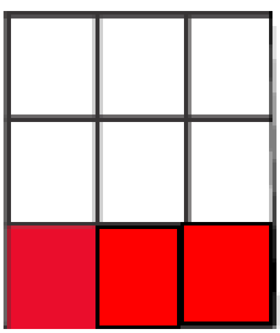
$\mathbf{1 : \frac{1}{9} = 9}$



$\mathbf{3 : \frac{1}{9} = 27}$



To divide a number (or a fraction) by a fraction we should multiply by its reciprocal $a : \frac{c}{d} = a \cdot \frac{d}{c}$



$$\frac{3}{9} : \frac{1}{3} = \frac{3}{9} : \frac{3}{9} = 1 = \frac{3}{9} x \frac{9}{3}$$

$$\frac{3}{9} : \frac{1}{18} = \frac{3\ x\ 18}{9\ x\ 18} : \frac{9\ x\ 1}{18\ x\ 9} = \frac{3\ x\ 18}{9\ x\ 1} = \frac{3}{9}\ x\ \frac{18}{1}$$

We brought both fractions to the same denominator “9·18”

Now we can just divide the numerators

When you do that, it turns out that we are multiplying the original fraction $\frac{3}{9}$ (dividend) by the reciprocal of the divisor $\frac{18}{1}$