

Factors

"Factors" are the numbers you multiply together to get another number:

$$\begin{array}{ccc} 2 \times 3 = 6 \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{Factors} \quad \text{Product} \end{array}$$

Factorization

Factorization is breaking(decomposing) a number by its factors. It is the same as representing a number as a product of two or more other numbers, for example:

- $40 = 4 \times 10$, $36 = 6 \times 6$
- $a \times b + a \times c = a \times (b + c)$
- $7 \times 5 + 7 \times 3 = 7 \times (5 + 3)$
- A natural number that is greater than 1 and has no positive divisors other than 1 and itself is called a prime number.
- Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else.

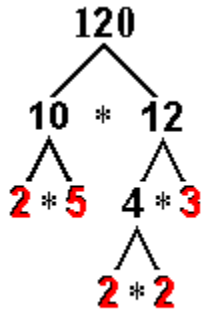
Prime numbers

Prime numbers are building blocks of all composite numbers. Prime number can only be divided by 1 or itself, so it cannot be factored any further. [Positive integers](#) other than 1 which are not prime are called [composite numbers](#).

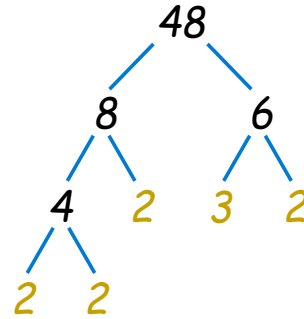
Prime factorization is finding which prime numbers multiply together to make the original number

There is only one (unique!) set of prime factors for any natural number.

Prime factorization process



$$2 \times 2 \times 2 \times 3 \times 5 = 120$$



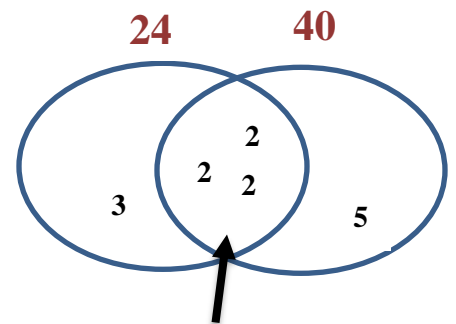
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Greatest Common Factor (Divisor)

The largest number that divides two given integers without remainder

To find GCF of two integers 24 and 40:

1. Prime factorize $24 = 6 \times 4 = 2 \times 3 \times 2 \times 2$ $40 = 4 \times 10 = 2 \times 2 \times 5 \times 2$
2. Find pairs $24 = 6 \times 4 = 2 \times 3 \times 2 \times 2$ $40 = 4 \times 10 = 2 \times 2 \times 5 \times 2$
3. Multiply them $2 \times 2 \times 2 = 8$
4. Both numbers are divisible by 8. $\text{GCF}(24, 40) = 8$



Greatest Common Factor

Word Problem 1



For Halloween the Jonson family bought 168 mini chocolate bars and 180 gummy worms. What is the **largest** number of kids between whom the Jonson family can divide both kinds of candy evenly?



To solve this problem we have to find the **Greatest Common Divisor (GCD)**, the **largest** number that can be a divisor for both 168 and 180.

Let's look at a set of all prime factors of 168 and 180.

For $168 = 2 \times 2 \times 2 \times 3 \times 7$. Any of these numbers as well as any of their products can be a divisor of 168.

For $180 = 2 \times 2 \times 3 \times 3 \times 5$.

These two sets have common elements. It means that both numbers are divisible by any of these common elements and any their products.

The greatest common factor (**GCF**) is the product of all common elements $2 \times 2 \times 3 = 12$. So the candy can be divided evenly between 12 kids.

$$168 \div 12 = 14$$

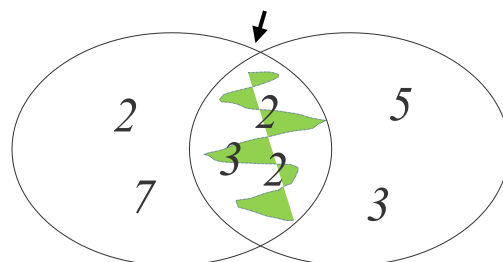
$$180 \div 12 = 15$$

Prime factors of 168 and 180 are $2 \times 2 \times 2 \times 3 \times 7 = 168$; $2 \times 2 \times 3 \times 3 \times 5 = 180$

GCF is the the common set of prime factors)

GCF is the intersection of the two sets of prime factors

Set of prime factors of 16



Set of prime factors of 180



Least Common Multiple (LCM)

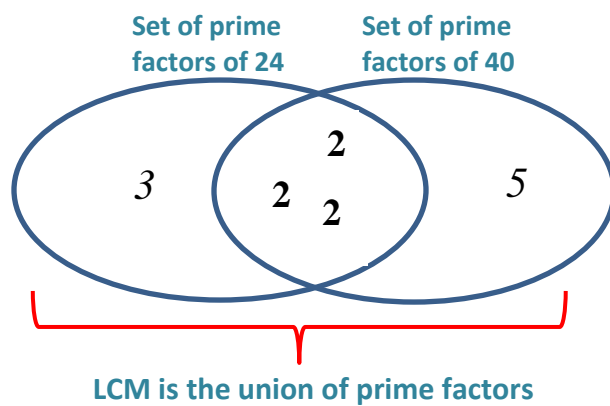
LCM is the smallest number that has given integers as factors. LCM is the smallest number that is a multiple of each of given integers.

To find LCM of two integers 24 and 40 you can find their multiples:

Multiples of 24	Multiples of 40
$24 \times 1 = 24$	$40 \times 1 = 40$
$24 \times 2 = 48$	$40 \times 2 = 80$
$24 \times 3 = 72$	$40 \times 3 = 120$
$24 \times 4 = 96$	$40 \times 4 = 160$
$24 \times 5 = 120$	$40 \times 5 = 200$

To find LCM you can also Prime factorize:

1. Prime factorize $24 = 6 \times 4 = 2 \times 3 \times 2 \times 2$ $40 = 4 \times 10 = 2 \times 2 \times 5 \times 2$
2. Multiply factors that represent each integer $2 \times 2 \times 2 \times 3 \times 5 = 120$
3. LCM (24, 40) = 120

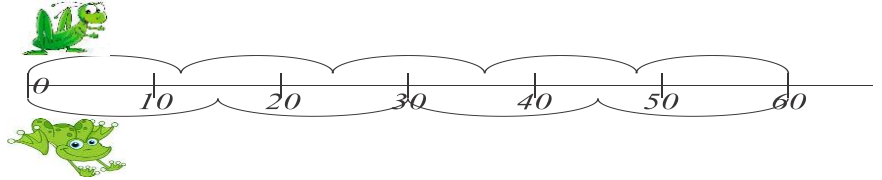


Word Problem 2

A grasshopper jumps the 12 cm distance each jump. A little frog jumps the 15 cm distance each jump. They start jumping from the point 0 and jump along a big ruler. What is the closest point (measure) on the ruler they both can land?

We should find the smallest number that is divisible by both 12 and 15 (i. e. LCM)

One method is to find multiples of 12 and multiples of 15



Multiples of 12	Multiples of 15
$12 \times 1 = 12$	$15 \times 1 = 15$
$12 \times 2 = 24$	$15 \times 2 = 30$
$12 \times 3 = 36$	$15 \times 3 = 45$
$12 \times 4 = 48$	$15 \times 4 = 60$
$12 \times 5 = 60$	$15 \times 5 = 75$
$12 \times 6 = 72$	$15 \times 6 = 90$

We also do it using prime factorization of 12 and 15: $12 = 2 \times 2 \times 3$ $15 = 3 \times 5$

$$\text{LCM: } 2 \times 2 \times 3 \times 5 = 60$$

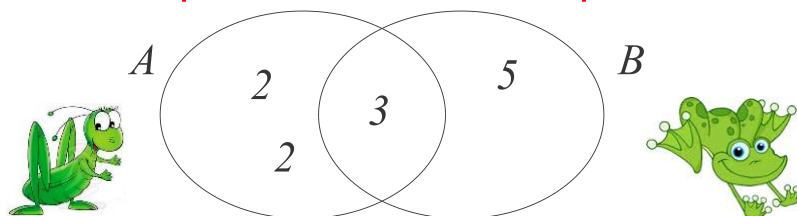
The number which we are looking for must be a product of prime factors of 12 AND 15, so it should be a **union** of two sets – set of prime factors of 12 and 15.

Set of prime factors of 12

LCM

Set of prime factors of 15

$$2 \times 2 \times 3 \times 5 = 60$$



60 is the smallest number, which is divisible by 12 and 15, LCM.

We found that **60** is the smallest number, which is divisible by 12 and 15, LCM. So, both the frog and the grasshopper will be eventually landing on the 60 mark on our ruler.