Math 4. Handout #4.

Algebra

Addition

Multiplication

$a+b=b+a$ ← Commutative property → $a \times b=b \times a$

$a+(b+c)=(a+b)+c$ ← Associative property → $a \times (b \times c)=(a \times b) \times c$

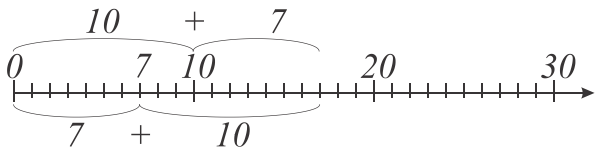
Distributive property

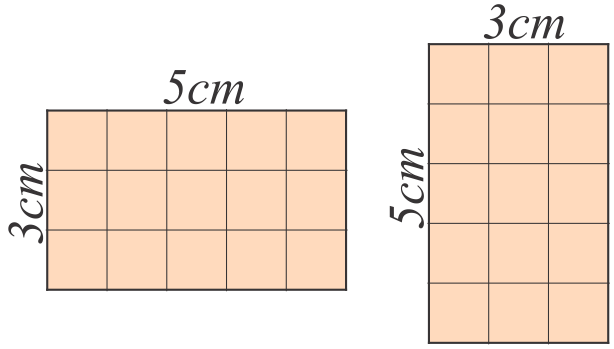
$a \times (b+c)=a \times b + a \times c$

$a \times (b - c)=a \times b - a \times c$

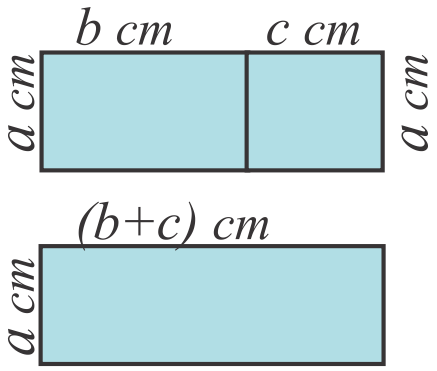
Commutative and associative properties are intuitively easy to understand.

Distributive property can be visualized as shown in the following figures:





$$S = 3cm \times 5cm = 5cm \times 3cm = 15cm^2$$



Look at the rectangles on the left:

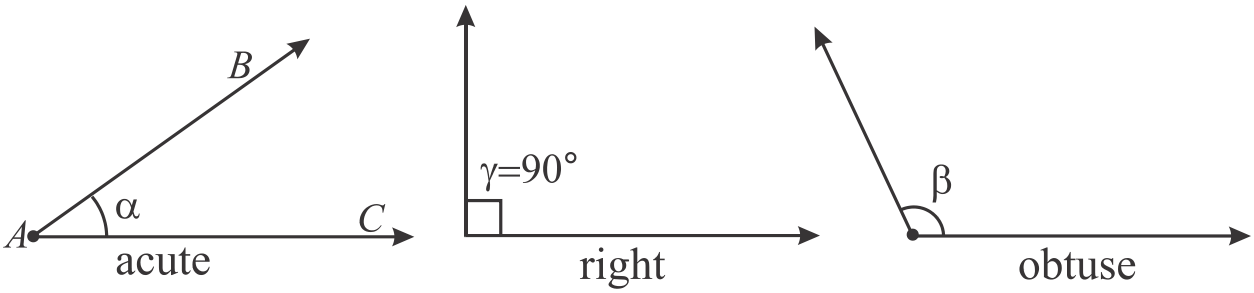
The combined area of these 2 rectangles is $S = a \times b + a \times c$

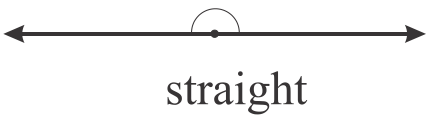
but the rectangle with one side a cm and the other $(b+c)$ cm will have exactly the same area.

Geometry.

An angle is the figure formed by two **rays**, sharing a common endpoint. The rays in the angle are called the **sides** and the common starting point of the rays is called the **vertex** of the angle

Angles notations are usually three capital letters with vertex letter in the middle: ∠BAC, or a small Greek letter: α. The measure of an angle is the amount of rotation required to move one side of the angle onto the other. Angles are named according to their measure:





Straight angle is formed by two rays on the same straight line and measures 180°.

- Two angles are **Adjacent** when they have a common vertex and a common side.

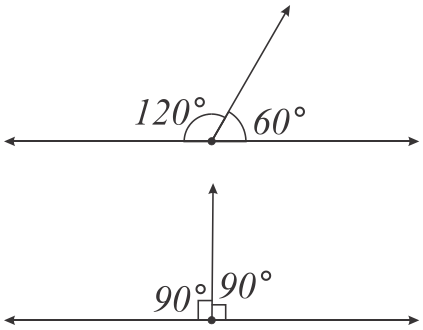
- Two Angles are **Supplementary** when they **add up to 180 degrees.**

  *Helpful note: "**S**" of **S**upplementary stands for "**S**traight" (180 degrees is a straight line)*

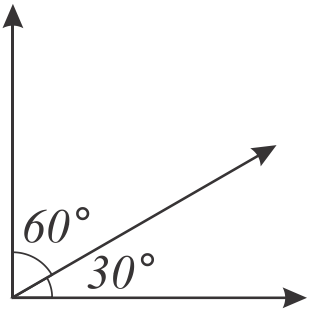
- Two angles are **Complementary** when they **add up to 90 degrees**

  Helpful note: "***C**" of **C**omplementary stands for "**C**orner"* ⌞ *(a Right Angle)*

- Right angle is supplementary to itself



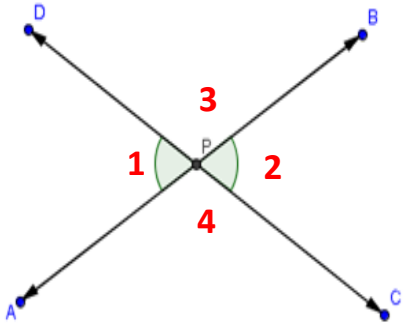
Supplementary



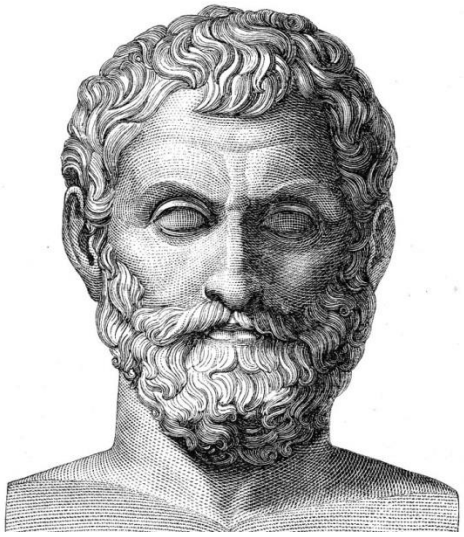
complementary

- Lines which intersect forming the right angle are perpendicular to each other.

- When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.

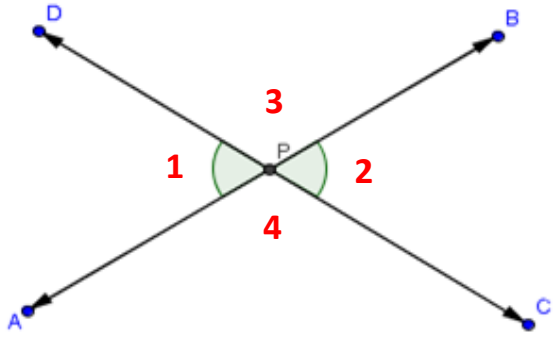


A historical legend tells us that when **Thales of Miletus (**624-546 BC), a Greek philosopher and mathematician, visited Egypt he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time. (Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).

Let's draw vertical angles: 1 and 2; 3 and 4 are two pairs of vertical angles.

Vertical angles theorem: **<u>Vertical angles are equal.</u>**

Proof:



$\angle 1 + \angle 3 = 180°$ because they are supplementary by construction.

$\angle 2 + \angle 3 = 180°$ because they are supplementary also by construction.

$\angle 1 + \angle 3 = \angle 2 + \angle 3$	Using the substitution property
$\angle 1 = \angle 2$	Using the subtraction property