MATH 10
ASSIGNMENT 22: CONIC SECTIONS
APRIL 7, 2019

Definition. Given two points F_1 and F_2 on the plane a distance $2c$ appart, the ellipse with focal points F_1 and F_2 and excentricity $0 < e = \frac{c}{a} < 1$ is the set of points for which the sum of the distances to F_1 and F_2 is $2a$.

Definition. Given two points F_1 and F_2 on the plane a distance $2c$ appart, the ellipse with focal points F_1 and F_2 and excentricity $e = \frac{c}{a} > 1$ is the set of points for which the difference of the distances to F_1 and F_2 is $2a$.

Definition. Given a point F and a line d on the plane a distance $p > 0$ appart, the parabola with focal point F and directrix d is the set of points which lie at equal distance from F_1 and d.

Remember, you should always draw pictures before you start calculating things in geometry!

Homework

1. *(Equations of conics)*
- (a) Show that a point (x, y) belongs to the ellipse of focal points $(-c, 0)$ and $(c, 0)$ and excentricity $e = c/a$ if, and only if, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for some fixed b. Find b in terms of a and c.
- (b) Show that a point (x, y) belongs to the hyperbola of focal points $(-c, 0)$ and $(c, 0)$ and excentricity $e = c/a$ if, and only if, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ for some fixed b. Find b in terms of a and c.
- (c) Show that a point (x, y) belongs to the parabola of focal point $(\frac{p}{2}, 0)$ and directrix $x = -\frac{p}{2}$ if, and only if, $y^2 = 2px$. What is the equation if instead the focal point is $(0, \frac{p}{2})$ and the directrix is $y = -\frac{p}{2}$?

2. In each case, recognize the conic, find the relevant points and lengths (example, focal points and excentricity) and sketch the curve:
- (a) $25x^2 + 16y^2 = 400$.
- (b) $\frac{(x-1)^2}{7} - \frac{(y-1)^2}{2} = 1$
- (c) $y^2 + 8x = 0$.

3. Find the distance between the point $(2, 4)$ and the line which passes through the points where the graph of the function $f(x) = x^2 + x$ intersects the graph of $f^{-1}(x)$.

***4.** Represent graphically the set of points (x, y) on the plane which satisfy $(y - x^2)(x + y - 2) \leq 0$.

5. Find the lines tangent to the ellipse $2x^2 + 3y^2 = 6$ which are parallel to the line $y = x$.

6. Find the line tangent to the parabola $y = x^2 - x - 2$ which is also perpendicular to the line $y = -\frac{x}{3}$. What is the common point between the parabola and the line you found?

***7.** Consider the *elliptic coordinates* on the plane, $x = \cosh a \cos b$, $y = \sinh a \sin b$. Here, $\cosh a = \frac{e^a + e^{-a}}{2}$ and $\sinh a = \frac{e^a - e^{-a}}{2}$.
- (a) Show that, for each fixed a, the above equations describe the points of an ellipse. Find the focal points and the excentricity of this ellipse.
- (b) Show that, for each fixed b, the above equations describe the points of a hyperbola. Find the focal points and the excentricity of this hyperbola.
- (c) Show that, at any point in the plane, the ellipse and the hyperbola defined by the above equations which pass though this point intersect at right angles.