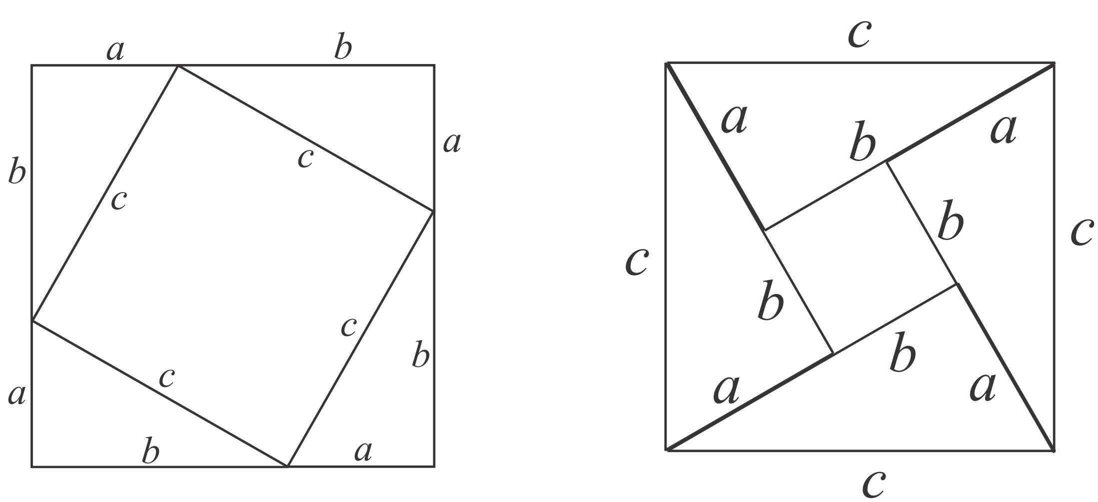
AM1. Class work 24.

Geometry.

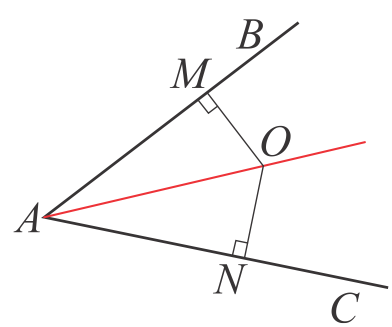
4 identical right triangles are arranged as shown on the picture. The area of the big square is
$S = (a + b) \cdot (a + b) = (a + b)^2$,
the are of the small square is
$s = c^2$.
The area of 4 triangles is
$4 \cdot \frac{1}{2} ab = 2ab$.
But also, can be represented as
$S - s = 2ab$
$2ab = (a + b) \cdot (a + b) - c^2 = a^2 + 2ab + b^2 - c^2 \Rightarrow \qquad a^2 + b^2 = c^2$



The **distance from a point to a line** is the length of the line segment which joins the point to the line and is perpendicular to the line.

Distance between the point and the line.

Theorem. All points of a bisector of an angle are located at an equal distance from both sides of the angle.



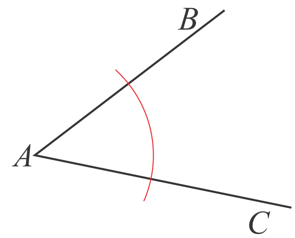
Ray $\overrightarrow{AO}$ is a bisector of the angle $\angle BAC$. Segment $[OM] \perp \overrightarrow{AB}$ and $[ON] \perp \overrightarrow{AC}$. We need to prove that $|OM| = |ON|$.

Angle $\angle MAO$ is equal to the angle $\angle OAN$, since the AO is bisector.

Angles $\angle AMO$ and $\angle ANO$ are both right angles, therefore angles

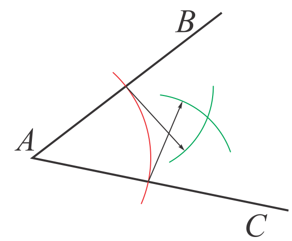
$\angle MOA$ and $\angle NOA$ are also equal and the segment $[AO]$ is the common side, two triangles ΔAMO and ΔANO are congruent by ASA criteria and $|OM| = |ON|$.

Construction problem:



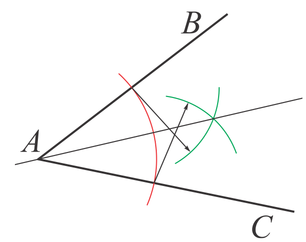
Draw the bisector of an angle.

1. Bisector is a median and an altitude in an isosceles triangle.
2. To construct a bisector mark two points on both sides of the angle on the same distance from the vertex of the angle.



3. With the same radius (greater than half of the length between two marked points) draw two arches with centers at marked points.

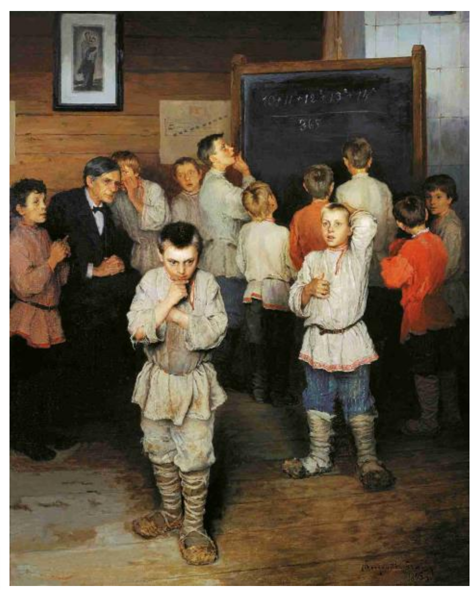
4. Draw a line through the vertex of the angle and the point of intersection of two arches.



Explain each step.

1. Compute:

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365} =$$



(a) $\dfrac{5x - 15}{x^2 - 9}$ **(b)** $\dfrac{a^2 - 5a + 6}{3a^2 - 6a}$

(c) $\dfrac{3x^2 + 14x - 5}{3x^2 + 2x - 1}$ **(d)** $\dfrac{5p - 15}{p^2 - 4}$

а) $a(x-y)+b(y-x)$; **б)** $x(a-b)+y(b-a)$;
в) $3(m-n)-a(n-m)$; **г)** $7a(a-b)-5(b-a)$;
д) $a(a-b)+4(b-a)$; **е)** $6(x-1)-x(1-x)$;
ж) $p(1-p)-3(p-1)$; **з)** $x^2(y-3)+7(3-y)$.