AM1. Class work 17.

Algebra.

Proportionality.

A car is driving with a constant speed 50 km/hour along a straight hideaway. How far it will go in 1 hour? 2 hours, 5 hours? 15 hours?

Fill the table:

Time (t)	1 hour	2 hours	5 hours	15 hours
Distance (d)				

What did you notice?

$$\frac{d_1}{t_1} = \frac{d_2}{t_2} = \frac{d_3}{t_3} = \frac{d_4}{t_4} = 50\ {}^{km}/_{h}$$

In other words, $d = 50\left({}^{km}/_{h}\right) * t(hour)$, 50 is a constant, and it's called a constant of proportionality. Longer travel ⇒ further from the initial point, and the ratio between these to variables will be always the same, *distance*:*time* is speed. If the time three time longer, the distance will be three time greater as well. (When the speed is constant of cause). Two variables, *distance* and *time,* are dependent from each other, they are in the relation of proportionality. Let's draw the graph in your notebooks, *abscissa* will be the time, and *ordinate* will be the distance.

We can write the relationship between the distance, time, and speed as

$$S = v \cdot t$$

A notebook cost 3 dollars. How much I need to pay if I buy 3 notebooks, 5, 12? What variables are used, what is the relationship between them. Can a constant of proportionality be found?

Two other problem.

1. A squirrel is doing a stock of acorns for winter. Every 20 minutes it brings 2 acorns. How many acorns it will have in 40 minutes? 80 minutes?

Time (t)	20 minutes	40 minutes	80 minutes	2 hours
Number of acorns				

2. Bacteria are dividing every 20 minutes. I want to make yogurt and I put 1 bacterium in a cup of milk. How many bacteria will be there in the milk in 20 minutes? in 40 minutes? in 80 minutes?

Time (t)	20 minutes	40 minutes	80 minutes	2 hours
Number of bacteria				

In which problem the variables are proportional?

Invers proportionality.

Let's go back to the first problem and take a look on it a little differently.

There two cities, A and B, distance between them is 10 km.

Time (t)	1 hour	2 hours	0.5 hours	0.1 hours
Speed (*v*)				

In this case we see an example of the inverse proportionality. If the speed is twice lower, we need two times longer time to get to the city B. If time is increased 2 times, speed is decreased 2 times.

Exercises.

1. Solve the following equations (hint: use proportionality)

$$\frac{x-\frac{2}{7}}{\frac{2}{7}}=\frac{48.3}{0.7}; \qquad \frac{1.8}{6.8}=\frac{0.042}{1\frac{1}{6}x+0.042}$$

2. The relationship between two variables is given in the table below. Is this relationship proportional? If so, what is the constant of proportionality?

a.

x	9	15	33	45	66
y	3	5	11	15	22

b.

x	3	2	5	4	6
y	9	4	25	16	36

c.

x	3	2	1	$\frac{1}{3}$	30
y	1	$\frac{3}{2}$	3	9	0.1

3. Are the following variables proportional ?
   a. Speed and time of movement on a distance of 50 km.
   b. Speed and corresponding distance after 2 hours of driving.
   c. Price of the 1 notebook and the number of notebooks which can be bought with 24 dollars.
   d. Length and the width of the rectangle with the area of 60 cm^2.
4. A car travel 60 km during a certain time. How this time will change, if the speed will be increased 3 times?

5. Which of the following formulas describe the direct proportionality, inverse proportionality or neither of the two?

$$P = 5.2b; \quad K = \frac{n}{2}; \quad a = \frac{8}{b}; \quad M = m:5; \quad G = \frac{1}{4k};$$
$$a = 8q + 1; \quad c = 4:d, \quad 300 = v \cdot t; \quad ab = 18; \quad S = a^2$$

6. Write as a single algebraic fraction.

$$\frac{x}{2} - \frac{y}{x}; \quad \frac{5}{2x} + \frac{y}{2y};$$

7. Reduce the following fractions (hint: $a^2 - b^2 = (a-b)(a+b)$):

$$\frac{3a^2 - a}{3a - 1}; \quad \frac{(a+4)^2}{a^2 - 4}$$

Geometry.

In geometry, two figures or objects are **congruent** if they have the same shape and size, or if one has the same shape and size as the mirror image of the other.[1]

More formally, two sets of points are called **congruent** if, and only if, one can be transformed into the other by a combination of **rigid motions**, namely a translation, a rotation, and a reflection. This means that either object can be repositioned and reflected (but not resized) so as to coincide precisely with the other object. So, two distinct plane figures on a piece of paper are congruent if we can cut them out and then match them up completely. Turning the paper over is permitted.

In elementary geometry the word *congruent* is often used as follows. The word *equal* is often used in place of *congruent* for these objects.

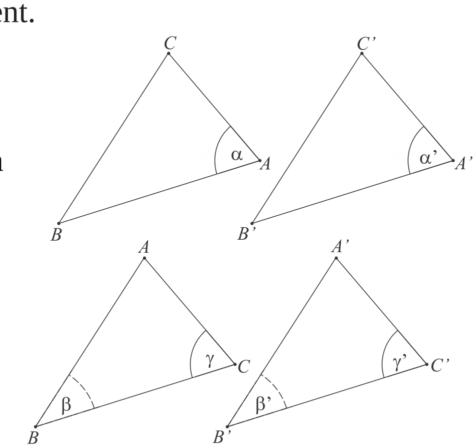
- Two line segments are congruent if they have the same length.
- Two angles are congruent if they have the same measure.
- Two circles are congruent if they have the same diameter.

Sufficient evidence for congruence between two triangles can be shown through the following comparisons:

- **SAS** (Side-Angle-Side): If two pairs of sides of two triangles are equal in length, and the included angles are equal in measurement, then the triangles are congruent.
- **SSS** (Side-Side-Side): If three pairs of sides of two triangles are equal in length, then the triangles are congruent.
- **ASA** (Angle-Side-Angle): If two pairs of angles of two triangles are equal in measurement, and the included sides are equal in length, then the triangles are congruent.

SAS (Side-Angle-Side).

ABC and A’B’C’ are two triangles such that $AC = A'C', AB = A'B'$, and $\angle A = \angle A'$ We need to prove that these triangles are congruent. Superimpose ΔABC onto $\Delta A'B'C'$ in such a way that vertex A would coincide with A', the side AC would go along $A'C'$, and side AB would lie on the same side of $A'C'$ as $A'B'$. Since AC is congruent to $A'C'$, the point C will merge with point C’., due to the congruence of $\angle A$ and $\angle A'$, the side AB will go along $A'B'$ and due to the congruence of these sides, the point B will coincide with B'. Therefor the side BC will coincide with $B'C'$.

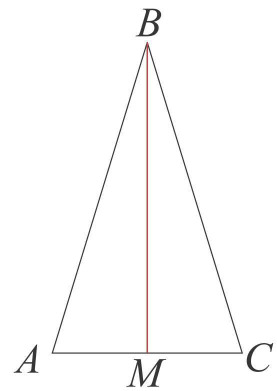


ASA (Angle-Side-Angle) ABC and $A'B'C'$ are two triangles, such that $\angle C = \angle C'$, $\angle B = \angle B'$, and $BC = B'B'$.

We need to prove, that these triangles are congruent. Superimpose the triangles in such a way that point C will coincide with point C', the side CB would go along $C'B'$ and the vertex A would lie on the same side of $C'B'$ as A'. Then: since CB is congruent to $C'B'$, the point B will merge with B', and due to congruence of the angle $\angle B$ an $\angle B'$, and $\angle C$ and $\angle C'$, the side BA will go along B'A', and side CA will go along C'A'. Since two lines can intersect only at 1 point, the vertex A will have merge with A'. Thus, the triangles are identified and are congruent.

Theorem. In isosceles triangle the bisector passed to the base (in isosceles triangle the base is the side different from two equal sides) is a median and an altitude as well.

Let the triangle ΔABC be an isosceles triangle, such that $AB = BC$, and BM is a bisector. We need to prove that BM is a median and an altitude, which means that $AM = MC$ and angle $\angle BMC$ is a right angle.



BM is a bisector, so $\angle ABM$ and $\angle MBC$, the triangle ΔABM is an isosceles triangle, so $AB = BC$ and the segment MB is common side for triangles ΔABM and ΔMBC. Based on the Side-Angle-Side criteria, the triangles ΔABM and ΔMBC are congruent. Therefore, $AM = MC$ (BM is a median), angles $\angle A$ and $\angle C$ are congruent. (Isosceles triangle has equal angles adjacent to the base).

$\angle A + \angle B + \angle C = 180° = 2\angle A + \angle B \Rightarrow 90° = \angle A + \frac{1}{2}\angle B$ but for the triangle ABM (as well as for MBC), $\angle A + \frac{1}{2}\angle B + \angle BMA = 180°$, therefore $\angle BMA = 90°$and BM is also an altitude.

Exercise.

On one side of an angle $\angle A$, the segments AB and AC are marked, and on the other side the segments $AB' = AB$ and $AC' = AC$. Prove that the lines BC' and $B'C$ met on the bisector of the angle $\angle A$.