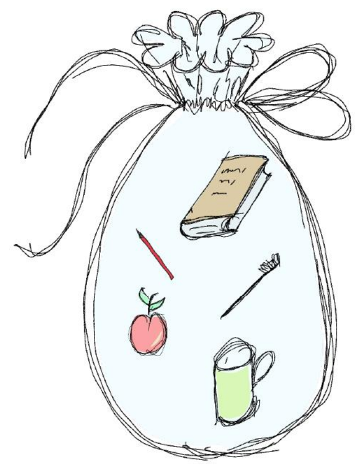
Accelerated Math 1. Class work 1. Algebra.

Sets.

I put a pencil, a book, a toothbrush, a coffee mug, and an apple into a bag. Do all these objects have something in common?



A set is a collection of objects that have something in common.

Can we call this collection of items a set? What is a common feature of all these objects? They are all in the bag, where I put them.

There are two ways of describing, or specifying, the members of a set. One way is by listing each member of the set, as we did with our set of things.

I can create, for example, a set of my favorite girls' names (F):

$\boldsymbol{F} = \{Mary,\ Kathrine, Sophia\}$.

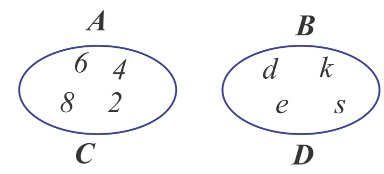
The name of the set is usually indicated by a capital letter, in my case is F, list of members of the set is included in curved brackets. I can also create a set of all girls' names (N):

$$\boldsymbol{N} = \{n|\ n = girl's\ name\} \qquad (1)$$

Of cause, I can't itemize all possible names, there are too many of them, we don't have enough space here for that, but I can describe the common feature of all members of the set – they are all girl's names. In the mathematical phrase above (1) I described the set, I call it N, which contains an unknown number of items, (we really don't know how many names exist), I use variable *n* to represent these items, girls' names. I show it by the phrase $n = girl's\ name$.

In our everyday life we use the concept of set quite often, we even have a special word for some sets, for example, the word "family" indicates a set of people, connected to each other, "class of 2020" – means all students who will graduate in 2020 and so on. Can you give more examples of such words and expressions?

Let’s consider two examples of sets:



Venn

Sets A and B are created by listing their items explicitly:

$\boldsymbol{A}$ = {2, 4, 6, 8}

$\boldsymbol{B}$ = {d, e, s, k}.

Sets C and D are created by describing the rules according with which they were created:

Two sets are equal if they contain exactly the same elements.

$\boldsymbol{C}$ is the set of four first even natural numbers.

$\boldsymbol{D}$ is the set of letters of the word "desk".

If we look closer on our sets $\boldsymbol{A}$ and $\boldsymbol{C}$, we can see that all elements of set $\boldsymbol{A}$ are the same as elements of set $\boldsymbol{C}$ (same goes for sets $\boldsymbol{B}$ and $\boldsymbol{D}$).

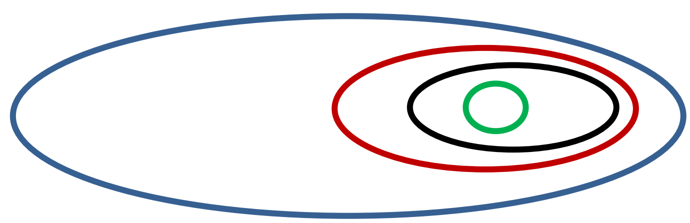
$$\boldsymbol{A} = \boldsymbol{C} \quad \text{and } \boldsymbol{B} = \boldsymbol{D}$$

If a set $\boldsymbol{A}$ contains element '2', then we can tell that element '2' belongs to the set $\boldsymbol{A}$. We have a special symbol to write it down in a shorter way: $2 \in \boldsymbol{A}$, $105 \notin \boldsymbol{A}$. **(**What does this statement mean?**)**

Whet a set is created, we can say about any possible element does it belong to the given set or not.

Let's define several sets:

> Set $\boldsymbol{W}$ will be the set of all words of the English language.
>
> Set $\boldsymbol{N}$ will be the set of all nouns existing in the English language.
>
> Set $\boldsymbol{Z}$ will be the set of all English nouns which have only 5 letters.
>
> Set $\boldsymbol{T}$={"table"}.



On a Venn diagram name all these sets:

If all elements of one set at the same time belong to another set then we can say that the first set is a subset of the second one. We have another special symbol to write this statement in a shorter way: $\subset$.

$$\boldsymbol{T} \subset \boldsymbol{Z} \subset \boldsymbol{Y} \subset \boldsymbol{W}$$

If set $\boldsymbol{V}$ is defined as a set of all English verbs, can you draw a diagram for set $\boldsymbol{V}$ on the picture above? Can you tell subset of which set $\boldsymbol{V}$ is?

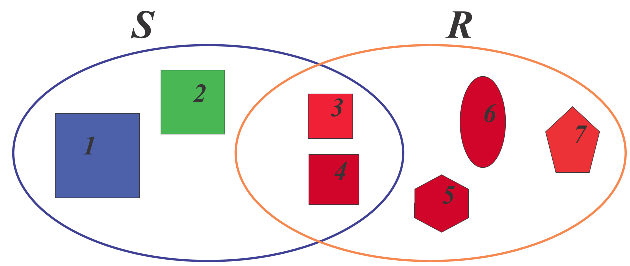
$\boldsymbol{V} \subset$ ____

$\boldsymbol{V} \not\subset$____

$\boldsymbol{V} \not\subset$____

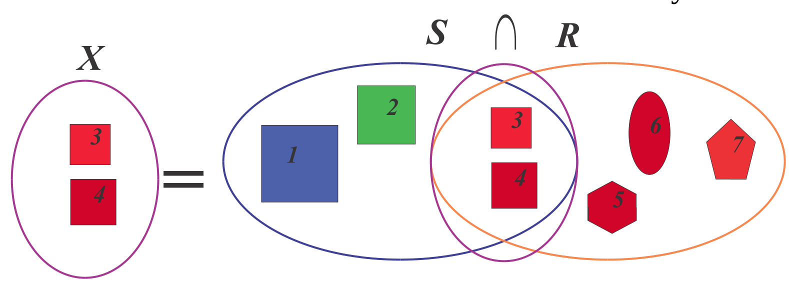
$\boldsymbol{V} \not\subset$____

When several sets are defined it can happened, that in accordance with all the rules we have implied, several objects can belong to several sets at the same time. For example, on a picture set $\boldsymbol{S}$ is a set of squares and a set $\boldsymbol{R}$ is a set of red shapes.



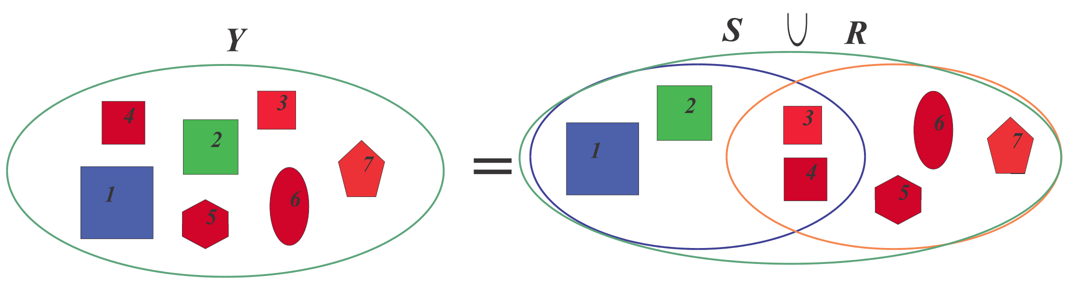
Figures 3 and 4 are squares and they are red, therefore they belong to both sets. Thus, we can describe a new set $\boldsymbol{X}$ containing elements that belong to the set $\boldsymbol{S}$ as well as to the set $\boldsymbol{R}$. The new set was constructed by determining which members of two sets have the features of both sets. This statement also can be written down in a shorter version by using a special symbol ∩. Such set $\boldsymbol{X}$ is called an **intersection** of sets $\boldsymbol{S}$ and $\boldsymbol{R}$.

$$\boldsymbol{X} = \boldsymbol{S} \cap \boldsymbol{R}$$



Another new set can be created by combining all elements of either sets (in our case $\boldsymbol{S}$ and $\boldsymbol{R}$). Using symbol ∪ we can easily write the sentence: Set $\boldsymbol{Y}$ contains all elements of set $\boldsymbol{S}$ and set $\boldsymbol{R}$:

$$\boldsymbol{Y} = \boldsymbol{S} \cup \boldsymbol{R}$$



Such set $\boldsymbol{Y}$ is called a **union** of set $\boldsymbol{S}$ and $\boldsymbol{R}$.

Set which does not have any element called an empty set (in math people use symbol ∅). For example, the set of polar bears living in the Antarctica is an empty set (there is no polar bears in Antarctica).

$\in$	element belongs to a set	$\not\subset$	one set is not a subset of another set
$\notin$	element does not belong to a set	$\cap$	intersection of two sets
$\subset$	one set is a subset of another set	$\cup$	union of two sets
$\varnothing$	empty set		

Exercise:

1. Give examples of several members of the following sets:

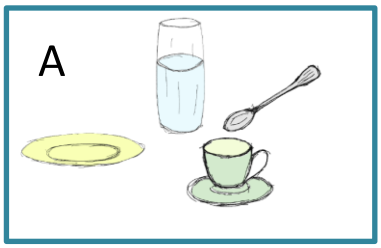
   Example:

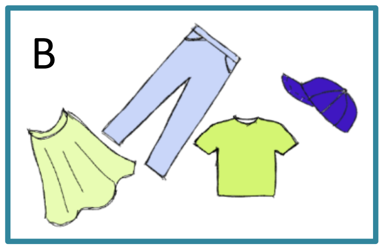
   $M = \{x \mid x = mammals\}$

   x can be a lion, a whale, a bat…

   a. $K = \{y \mid y = letter\ of\ english\ alfabet\}$
   b. $M = \{x \mid x = flower\}$
   c. $X = \{m \mid m = even\ number\}$
   d. $P = \{k \mid k = color\}$ (2)

2. Which word we can use to describe a set, subset of which is drawn on the pictures below:



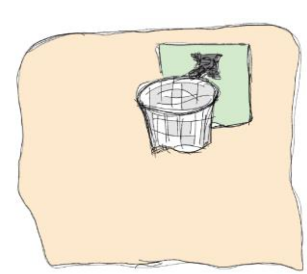


   List a few other members of these sets. (2)

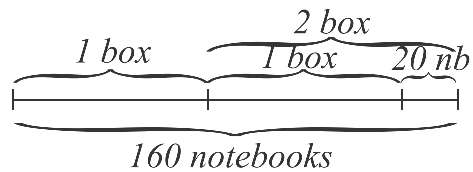
3. $A = \{2, 5, 0, 1\}$, $B = \{2, 0, 1\}$, $C = \{0, 2, 5, 1\}$, and $D = \{2, 0, 5, 4, 1\}$
   Which sets are equal? (2)

4. There are 21 students in a Math class. 10 students like apples and 15 students like pears.
   a. Show that there are some students, who like both apples and pears.
   b. Is it possible to determine if there any students who do not like apples and do not like pears? Explain your answer.
   c. Assume, that each student likes at least one of the fruits. (This means that each student likes either apples, or pears, or both). How many do students like both pears and apples?

5. The same Math class (with 20 students) forms a soccer team and a basketball team. Every student signs up for at least one team: 12 students play only soccer; 2 students play both soccer and basketball; How many students play basketball only? (6)



6. Students who participated in math coopetition had to solve 2 problems, one in algebra and another in geometry. Among 100 students 65 solved algebra problem, 45 solved geometry problem, 20 students solved both problems. How many students didn't solve any problem at all? (6)
7. 240 students from New-York and Seattle attended a math camp. Of the total number of students, 125 were boys. 65 boys were from New-York. There were 53 girls from Seattle. How many students came from New-York? (4)
8. In 2 boxes there are 160 notebooks altogether. In one box there are 20 more notebooks than in the other. How many notebooks are there in each box? Solve the problem using the schematic picture on the right. (4)



9. Father is three times as old as his son. How old are they, if their combine age is 48? (draw the schematic picture similar to the picture in the previous problem). (4)
10. Viktor has 2 more sisters then brothers. How many boys and girls are there in the Victor's family, if they have 5 kids altogether? If they have 7 kids altogether? (4)
11. Place parentheses into the following expression so that the statement is true.
    a. $15 - 35 + 5 \div 4 = 5$
    b. $60 + 40 - 16 : 4 = 66$
    c. $24 : 56 - 8 \cdot 4 = 1$
    d. $96 - 12 \cdot 6 : 3 = 8$
    e. $64 : 64 - 8 \cdot 4 = 2$
    f. $63 : 9 + 54 = 1$
    g. $75 - 15 : 5 + 10 = 22.$ (6)
12. John came to a lemonade stand with a big empty pitcher which can hold 5 liters of lemonade. He wanted to buy only 1 liter of lemonade, but a merchant had jars which can hold 3 liters and 2 liters of liquid. How merchant can measure 1 l. of lemonade if jars do not have any marks on them? Next time when John came to the stand with exactly the same pitcher, the merchant had only 3 l and 5 l jars. Can he sell to John exactly 4 l of lemonade? (6)
13. Write the expression for the following statements:

    Example:

    Mary is a years old. Richard is 2 years older. What is the Richard's age?

    Answer: $a + 2$

    a. Lina is b years old, David is twice as old as Lina. What is the David's age?

b. John solved x math problems, Julia solved twice as many as John did, and Sam solved 2 problems fewer than John did. How many problems they all solved together? (4)

14. Compute:

a. $\dfrac{1-\dfrac{7}{12}}{10}$;

c. $\dfrac{2+\dfrac{1}{4}}{3}$;

b. $\dfrac{12}{1-\dfrac{1}{4}}$;

d. $\dfrac{5}{4-\dfrac{2}{3}}$

(6)

15. The cyclist rode a bicycle at a speed of 14 km / h, and the distance he traveled is 6 km. How much time did it take? Represent the answer in minutes. (4)

16. Solve the following riddles (each letter represents a digit)

$$\begin{array}{r} \text{SEND} \\ +\ \text{MORE} \\ \hline \text{MONEY} \end{array} \qquad \begin{array}{r} \text{EAT} \\ +\ \text{THAT} \\ \hline \text{APPLE} \end{array} \qquad \begin{array}{r} \text{CIRCLE} \\ \text{CIRCLE} \\ +\ \text{CERCLE} \\ \hline \text{SPHERE} \end{array} \qquad \begin{array}{r} \text{ELF} \\ +\ \text{ELF} \\ \hline \text{FOOL} \end{array}$$

(The first one is a classic, published in the July 1924 issue of Strand Magazine by Henry Dudeney.)

(8)