


# CONVERSION OF UNITS

# ORDERS OF MAGNITUDE

# Conversion of Units

- **For <u>the same quantity measured</u>, we can convert units using an equivalence statement which shows the relationship between the units (this relationship is called a *conversion factor*).**

**Imperial-Metric equivalence statements:**

| Units of Length | Units of Weight | Units of Capacity |
|---|---|---|
| 1 in = 2.54 cm | 1 oz = 28.35 g | 1.06 qt = 1 L |
| 3.28 ft = 1 m | 1 lb = 454 g | 1 gal = 3.79 L |
| 1 mi = 1.61 km | 2.2 lb = 1 kg | |

- **Units that measure *physical quantities* (like the examples above ) always have a common zero.**
- **<u>Within the Metric System</u> itself, by design, conversion factors are always a power of 10.**

# Dimensional Analysis

- **<u>Dimensional Analysis</u>** (also called *Factor-Label Method* or the *Unit Factor Method*) **is a problem-solving method that uses the fact that any number or expression can be <u>multiplied by one</u> *(Magic One)* without changing its value.**
- **To help with conversion of units, Magic One is built using the equivalence statement:**

| Equivalence Statement(s) | | Magic One(s) | |
|---|---|---|---|
| 1 in = 2.54 cm | ⟹ | $\frac{1\ \text{in}}{2.54\ \text{cm}} = 1$ | $\frac{2.54\ \text{cm}}{1\ \text{in}} = 1$ |
| 2.2 lb = 1 kg | ⟹ | $\frac{2.2\ \text{lb}}{1\ \text{kg}} = 1$ | $\frac{1\ \text{kg}}{2.2\ \text{lb}} = 1$ |

# **Example:** Convert 130 lbs to kg

- **<u>Step 1</u>. Write the *original* measurement as a *unit fraction*:**

130 lbs / 1

- **<u>Step 2</u>. Using the equivalence statement, build a *magic one* (<u>building rule</u> - the *numerator unit* is the unit you *want*, the *denominator unit* is the *original* unit you want to *eliminate*):**

$$2.2 \text{ lb} = 1 \text{ kg} \Longrightarrow \frac{1 \text{ kg}}{2.2 \text{ lb}} = 1$$

- **Step 3: multiply your unit fraction by your magic one and write your answer in the new units:**

$$\frac{130 \cancel{\text{lbs}}}{1} \cdot \boxed{\frac{1 \text{ kg}}{2.2 \cancel{\text{lbs}}}} = \frac{130 \text{ kg}}{2.2} = 59.1 \text{ kg}$$

**Example:** The fuel tank of a plane can hold 876 liters of gas. How many gallons would it be?


Equivalency: 1 gallon = 3.8 liters

$$\frac{876\ \cancel{L}}{1} \cdot \boxed{\frac{1\ gal}{3.8\ \cancel{L}}} = \frac{876\ gal}{3.8} = \mathbf{230.5\ gal}$$

**Exercise:** As a practical joke, on the show Candid Camera, a gas station listed their price as **$1.79/L**. People gassing up thought they were getting a great deal, but then were outraged when their total owed came up. **WHY?**

What should we do?

# Let’s carefully examine:

“Listed their price as $1.79/L”

Equivalency: 1 gal = 3.79 L

$$\frac{\$1.79}{1\,\cancel{\text{L}}} \cdot \boxed{\frac{3.79\,\cancel{\text{L}}}{1\text{ gal}}} = \frac{\$6.78}{1\text{ gal}}$$

“The deal” was actually **$6.78/gal**!

# Conversion of Temperature

**When converting temperature between different scales, we need to pay attention to the fact that they all have <u>different "0" points</u>, therefore not only a** ***multiplication factor*** **is needed but also a** ***shift*****.**

**Kelvin**

$$K = {}^\circ C + 273.15$$

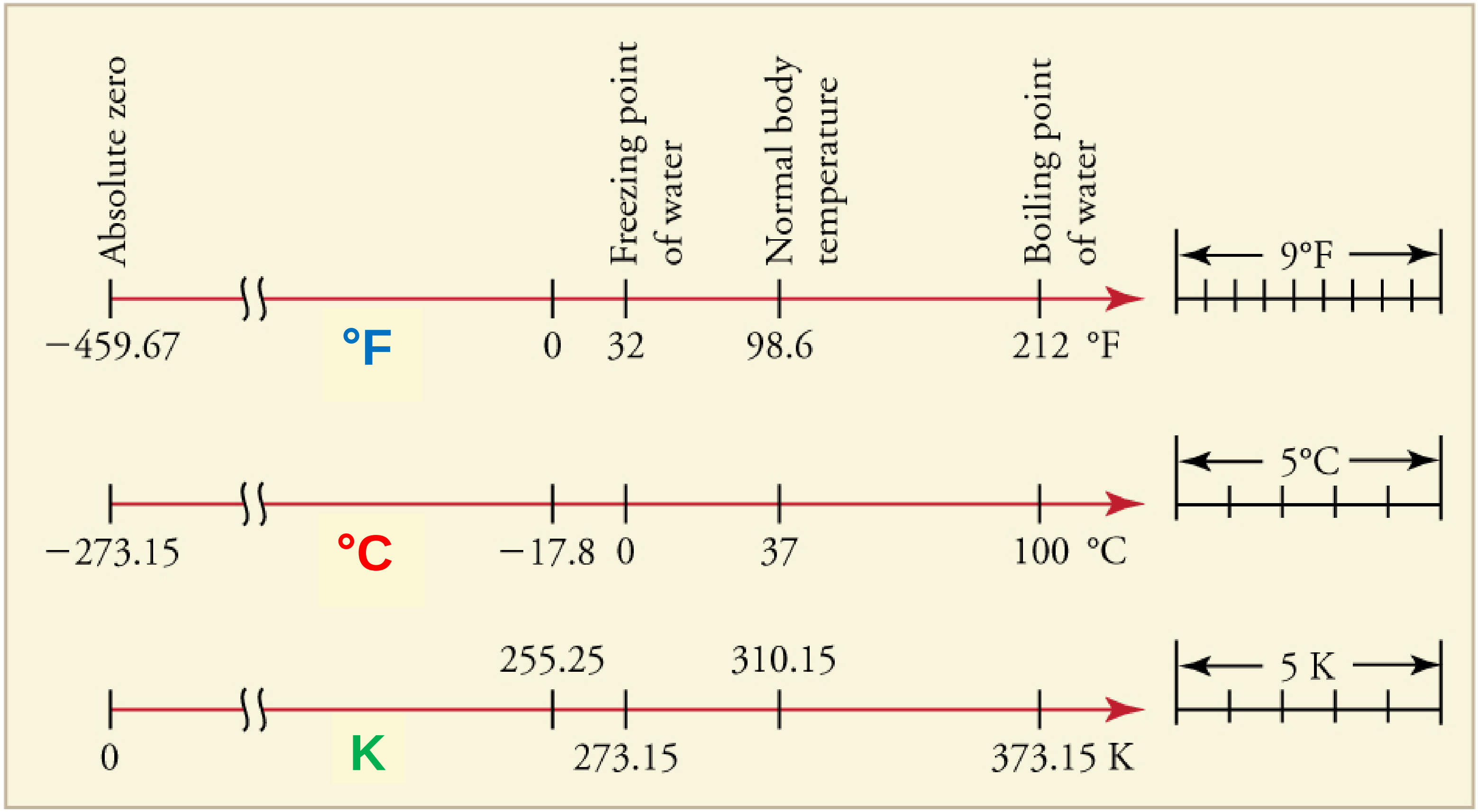
**Fahrenheit**

$$^\circ F = {}^\circ C \cdot 1.8 + 32 = {}^\circ C \cdot \frac{9}{5} + 32$$

**Celsius**

$$^\circ C = \left({}^\circ F - 32\right)/1.8 = \left({}^\circ F - 32\right) \cdot \frac{5}{9}$$

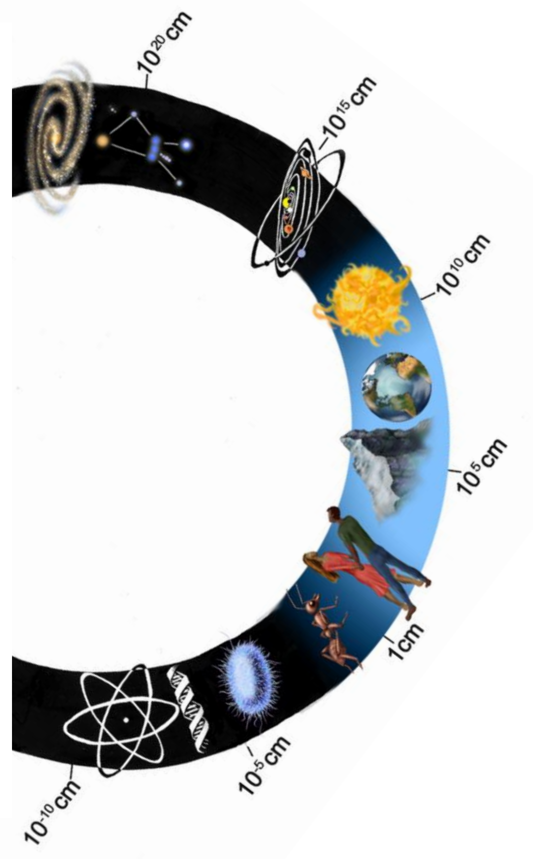
# Temperature Scales




**Note: according to the latest research, <u>normal human body temperature</u> is 36.8 °C ±0.7 °C, or 98.2 °F ±1.3 °F.**

# Orders of Magnitude

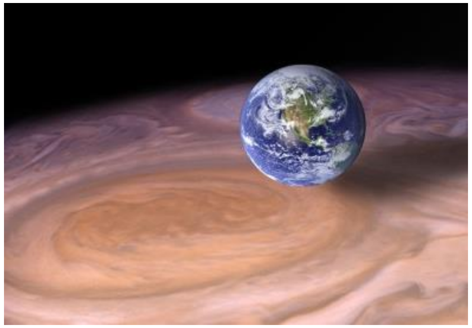
- **<u>Orders of magnitude</u>** are numbers on a scale where each number is rounded to the ***nearest power of ten***.
- Orders of magnitude are generally used to make **very approximate comparisons** of measurements, and reflect **very large differences**.



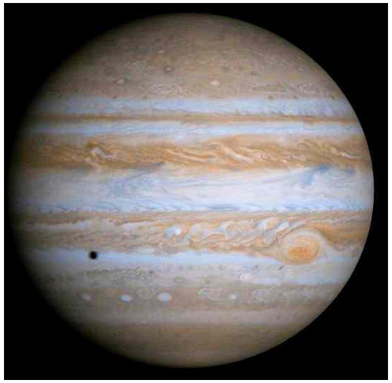

- To be able to **compare** something by means of orders of magnitude we have to **use the same units** (Standard SI units are typically used)!

# Examples of Difference

- If two numbers differ by **one order of magnitude**, one is about **ten** times larger than the other.



$\times\, 10 \approx$



- If they differ by **two orders of magnitude**, they are related by a factor of about **100**.

$\times\, 100 \approx$

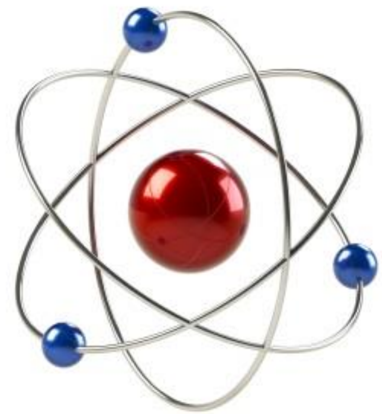
# By how many orders of magnitude is a giraffe taller than an ant?

***A giraffe is about 6 m tall:***
*nearest power of ten is*
*10 m = 1x10$^{1}$ m =* ***10$^{1}$ m***

***An ant is about 0.7 mm tall:***
*nearest power of ten is*
*1 mm = 1x10$^{-3}$ m =* ***10$^{-3}$ m***

**The giraffe is taller by 1-(-3)=4 four orders of magnitude.**

# *By how many orders of magnitude is human bigger than an atom?*



***A human is about 175 cm tall:***
*nearest power of ten is*
*100 cm = 1 m =* ***$10^0$ m***

***An atom is about 0.1 nm:***
*nearest power of ten is*
*0.1 nm = $0.1 \times 10^{-9}$ m =* ***$10^{-10}$ m***

**The human is bigger by 0-(-10)=10 ten orders of magnitude.**

# Blue Whale heart and Human heart

**A Blue Whale heart is about 2000 lb:**

*converting lb to kg* $2000\ lb \times \frac{1\ kg}{2.2\ lb}$ = 909 kg

*nearest power of ten is 1000 kg =* ***$10^3$ kg***

**A human heart is about 250 g:**

*converting g to kg 250 g = 0.25 kg*

*nearest power of ten is*

*0.1 kg =* ***$10^{-1}$ kg***

***Difference: 3-(-1) = 4***

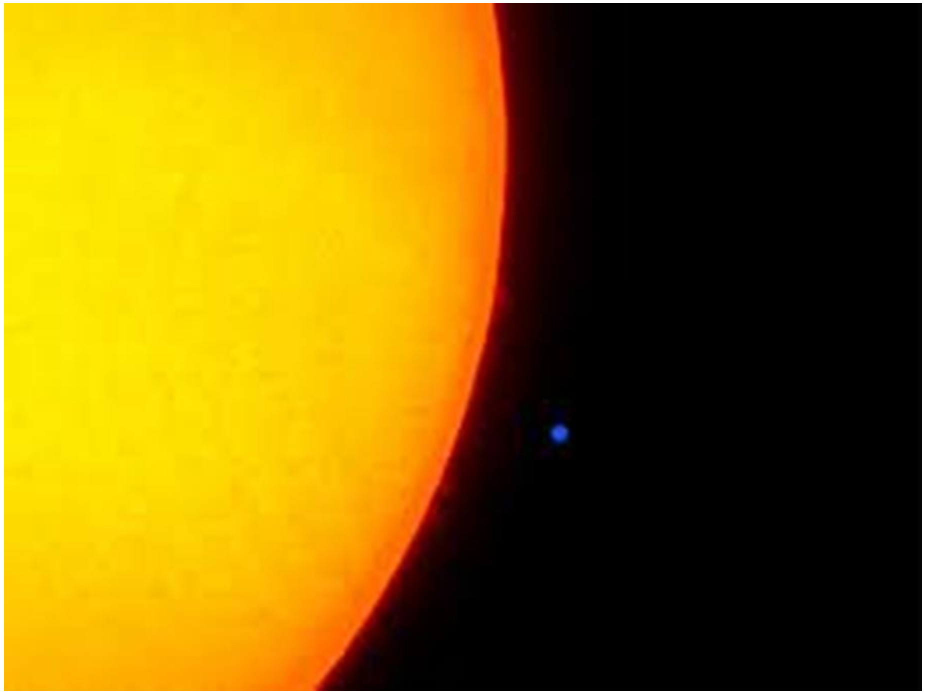
**four orders of magnitude**

# *Let us compare Sun and Earth in terms of orders of magnitude*

| | **Sun** | **vs** | **Earth** |
|---|---|---|---|
| • Mass | $10^{33}$ g | | $10^{27}$ g |
| • Radius | $10^{9}$ m | | $10^{7}$ m |

**Sun is heavier than Earth by 6 orders of magnitude and bigger by 2 orders of magnitude.**

**Can you imagine that difference?**



# Examples

| Order of Magnitude of some Masses | | Order of Magnitude of some Lengths | |
|---|---|---|---|
| **MASS** | **grams** | **LENGTH** | **meters** |
| electron | $10^{-27}$ | radius of proton | $10^{-15}$ |
| proton | $10^{-24}$ | radius of atom | $10^{-10}$ |
| virus | $10^{-16}$ | radius of virus | $10^{-7}$ |
| amoeba | $10^{-5}$ | radius of amoeba | $10^{-4}$ |
| raindrop | $10^{-3}$ | height of human being | $10^{0}$ |
| ant | $10^{0}$ | radius of Earth | $10^{7}$ |
| human being | $10^{5}$ | radius of Sun | $10^{9}$ |
| pyramid | $10^{13}$ | Earth-Sun distance | $10^{11}$ |
| Earth | $10^{27}$ | radius of Solar System | $10^{13}$ |
| Sun | $10^{33}$ | distance from Sun to nearest star | $10^{16}$ |
| Milky Way galaxy | $10^{44}$ | radius of Milky Way galaxy | $10^{21}$ |
| the Universe | $10^{55}$ | radius of observable Universe | $10^{26}$ |

# Powers of Ten video

https://www.youtube.com/watch?v=bhofN1xX6u0

https://www.youtube.com/watch?v=EMLPJqeW78Q