

Assignment 17: The Elephant-Hamster Duality Part II

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The Axioms

- **Elephant Friendship Axiom.** Anytime n elephants are associated to a single hamster, the elephants make friends such that each elephant has exactly two friends in the group (friendship is mutual). Each friendship pair has an associated theme song. Of the n total friendship pairs, the set of n associated theme songs is the same as the set of the n theme songs that are formed by the set of n elephants associated to a single hamster. It is possible to remove an elephant from the group: if this happens, that elephants friends become friends with each other and the two songs associated with the departing elephant combine to make the theme song of this new friendship pair.
- **Family Drama Axiom.** Given elephants A, B in the same family and elephant C outside the family, let H, J be the hamsters associated to the elephant pairs (A,C) and (B,C) respectively. Then the pair of theme songs around hamster H is the same as the pair of theme songs around hamster J .

Axiom Problems!

Using the above axioms, answer the following questions:

1. Consider elephant A is associated to hamster H but is *not* associated to hamster J. Let B be the elephant in the same family as A that *is* associated to hamster J, which we find by using the Family Drama Axiom.

Is there an axiom that guarantees that there will be an elephant associated to both hamsters H and J?

Let C be an elephant associated to both hamsters H and J. Determine which hamster A and C have in common (i.e., which hamster they are both associated to).

Elephant C has a unique hamster in common with A and thus a pair of corresponding theme songs with elephant A. Similarly, C also has a pair of theme songs with elephant B which we find through the hamster that B and C have in common. Must these two pairs of theme songs be the same?

2. Let elephants A, B, and C be associated to hamster H. Consider just elephants A and B and their association to hamster H. This association has two corresponding theme songs, from the Theme Song Existence Axiom. Let [AB] denote this pair of theme songs, whatever they are. Similarly define [AC] and [BC] as pairs of theme songs. Finally let [ABC] denote the set of three theme songs formed from elephants A, B, and C all associated to H. Use the Elephant Friendship Axiom to prove that [ABC] contains one song from [AB], one from [AC], and one from [BC].

Troop. Here is one definition we will be using: a set of n elephants and n hamsters arranged so that each elephant is associated to exactly two of the hamsters and each hamster is associated to exactly two of the elephants is called a *troop*. Any pair of elephants in the troop whose common associated hamster is also in the troop is called a pair of companions; also, since each hamster associated to its pair of elephants has two theme songs, we say that the total set of all such songs from the hamsters in the troop is called the troops set of songs.

A troop with n elephants and n hamsters is called a troop of size n ; as a special case, a troop of size 3 is called a *crew*.

3. Prove that in any crew, there are three songs among its set of songs that combine to the spongebob theme.
4. Prove that if elephants A and B with associated hamster H have jingle bells as both their theme songs, and elephants B and C with distinct hamster J also have jingle bells as both their theme songs, then elephants A and C are in the same family.
5. Prove that if a troops set of songs consists of only one song, then that troop has size 4, and its elephants come in two family pairs.