**Definition 1.** The Fibonacci numbers is the sequence of numbers constructed by the following rule: $F_1 = F_2 = 1$ and for $n > 1$, $F_{n+1} = F_n + F_{n-1}$. Here are the first several Fibonacci numbers: $1, 1, 2, 3, 5, 8, 13, 21, \ldots$. (Some people also define $F_0 = 0$.)

HOMEWORK

1. Solve the following equations and inequalities:

   (a) $\dfrac{x+1}{x+2} < 5$ (b) $\sqrt{x+4} = x + 2$ (c) $\dfrac{1}{x} = x - 2$ (d) $x^2 - 5x + 6 > 0$

2. Consider the sequence of circles inscribed in a $60°$ angle as shown below.
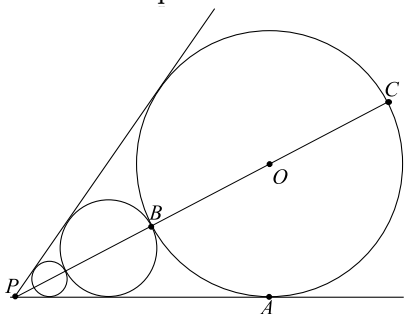
   (a) Prove that radii of these circles form a geometric progression, and find the common ratio r (hint: if $PO = x$, what is PA? PB?).
   (b) If the picture contains 10 circles, and the smallest circle has radius 1, what is the total area of all these circles?

3. Find the sum $1 + 11 + 111 + \cdots + 11\ldots11$, where the last number contains 57 ones. [Hint: find the sum $9 + 99 + 999 + \cdots + 99\ldots99$ and divide by 9.]

4. On Halloween, Mark has collected 779 pieces of candy. If he starts eating them on November 1st, eating one piece on the first day, two pieces on the second day, three pieces on the third day and so on, how long will his candy last?

5. [attributed to certain Leonardo of Piza, also called Fibonacci, 1202]

   Somebody buys a pair of rabbits and places them in a pen. The nature of rabbits is such that each month a pair of rabbits gives birth to another pair, and they start reproducing upon reaching the age of 2 months.
   How many pairs of rabbits will he have in one year (considering the rabbits immortal)?

month		number of pairs
1	🐇	1
2	🐇	1
3	🐇🐇	2
4	🐇🐇🐇	3
5	🐇🐇🐇🐇🐇	5

6. Show that for any n, $F_1 + F_2 + \ldots F_n = F_{n+2} - 1$. [Hint: $F_{n+2} = F_{n+1} + F_n$.]

7. (a) Which of Fibonacci numbers are even? Find the pattern and try to explain why this patterns holds.
   (b) Which of Fibonacci numbers are divisible by 3? Find the pattern and try to explain why this patterns holds.