MATH 7, ASSIGNMENT 18: SNAKE METHOD

Today we were talking about equations, including the ones with absolute values, and the snake method for solving inequalities. A few sample problems are given below.

1. Solve equation $|3x - 5| = 10$

Solution: To solve this equation we need to consider two cases, the one when $3x - 5 \geq 0$ and the one when $3x - 5 < 0$.

Case 1. $3x - 5 > 0$. In this case, $|3x - 5| = 3x - 5$, and the equation can be rewritten as

$$3x - 5 = 10.$$

We can easily solve this equation, getting $x = 5$. Substituting this value to the equation, we can see that it satisfies it.

Case 2. $3x - 5 < 0$. In this case, $|3x - 5| = -(3x - 5) = -3x + 5$, and the equation can be rewritten as

$$-3x + 5 = 10.$$

We can easily solve this equation, getting $x = -\frac{5}{3}$. Substituting this value to the equation, we can see that it satisfies it.

Therefore, there are two solutions to the equation: $x = 5, -\frac{5}{3}$.

2. Solve inequality $|x - 4| < 7$.

Solution: Again, as before, we need to consider two cases, the one when $x - 4 \geq 0$ and the one when $x - 4 < 0$.

Case 1. $x - 4 \geq 0$ means that $x \geq 4$. Now, since $x - 4 \geq 0$, we have $|x - 4| = x - 4$, and the inequality can be rewritten as

$$x - 4 < 7$$

Solving this inequality gives us $x < 11$. But remember, x must be greater than or equal to 4! So, combining both things together, we get $4 \leq x < 11$, or $x \in [4; 11)$.

Case 2. $x - 4 < 0$ means that $x < 4$. Now, since $x - 4 < 0$, we have $|x - 4| = -(x - 4) = 4 - x$, and the inequality can be rewritten as

$$4 - x < 7$$

Solving this inequality gives us $x > -3$. But remember, x must also be less than 4! So, combining both things together, we get $-3 < x \leq 4$.

Combining Cases 1 and 2 together, we get the final solution to the inequality: $-3 < x < 11$ or

$$x \in (-3, 11)$$

3. Solve the inequality $(x + 1)(x - 2)^2(x - 4)^3 \leq 0$.

Solution: Notice that if we solve the corresponding equation $(x + 1)(x - 2)^2(x - 4)^3 = 0$, we get $x = -1, 2, 4$. Therefore, we need to consider the following 4 intervals: $(-\infty; -1), (-1; 2), (2; 4), (4; \infty)$.

Notice that in the 1st interval, the expression $(x + 1)(x - 2)^2(x - 4)^3$ is negative, and therefore satisfies the inequality.

Then, as x "crosses" point 1, the expression changes its sign to '$+$', and therefore the interval $(-1; 2)$ does not satisfy the inequality.

Now, crossing point 2 again won't change the sign of the expression, because $(x - 2)^2$ is always positive. Therefore, the interval $(2; 4)$ also doesn't satisfy the inequality.

Finally, crossing point 4, the expression changes its sign to '–', and therefore the interval $(4; \infty)$ satisfies the inequality. So, the answer to the inequality is:

$$x \in (-\infty; -1] \cup 2 \cup [4; \infty)$$

The method used to solve this problem is called a "snake method."

HOMEWORK

1. Solve the following equations.

(a) $|x-3|=5$

(b) $|2x-1|=7$

(c) $|x^2-5|=4$

2. Solve the following equations.

(a) $\dfrac{(x+1)}{(x-1)}=3$

(b) $\dfrac{(x^2-9)}{(x+1)}=(x+3)$

(c) $x-\dfrac{3}{x}=\dfrac{5}{x}-x$

3. Solve the following inequalities, show solution on the real line, write the answer in the interval notation.

(a) $|x-2|>3$

(b) $|x-1|>x+3$

(c) $\dfrac{(x-2)}{(x+3)}\leq 3$

4. Solve the following inequalities, using the snake method. Show solution on the real line. Write the answer in the interval notation.

(a) $(x-1)(x+2)>0$

(b) $(x+3)(x-2)^2\leq 0$

(c) $x(x-1)(x+2)\geq 0$

(d) $x^2(x+1)^5(x+2)^3>0$

***5.** Solve the following inequalities, using the snake method. Show solution on the real line. Write the answer in the interval notation.

(a) $|x^2-x|\geq 2x$

(b) $\dfrac{x(x-1)^2}{(x+1)^2}\geq 0$