MATH 7
ASSIGNMENT 17: QUADRATIC EQUATION CONTINUED

VIETA FORMULAS

If an equation $p(x) = 0$ has root a (i.e., if $p(a) = 0$), then $p(x)$ is divisible by $(x - a)$, i.e. $p(x) = (x - a)q(x)$ for some polynomial $q(x)$. In particular, if x_1; x_2 are roots of quadratic equation $ax^2 + bx + c = 0$, then $ax^2 + bx + c = a(x - x_1)(x - x_2)$.

Therefore, if $a = 1$, then

$$\begin{aligned} x_1 + x_2 &= b \\ x_1 x_2 &= c \end{aligned}$$

HOMEWORK

1. Let a and b be some numbers. Use the formulas discussed in previous classes to express the following expressions using only $(a + b) = x$ and $ab = y$.
Example: Let's express $a^2 + b^2$ using only $a + b$ and ab. We know that $(a + b)^2 = a^2 + 2ab + b^2$. From here, we get:

$$a^2 + b^2 = (a + b)^2 - 2 \times ab = x^2 - 2 \times y$$

(a) $(a - b)^2$
(b) $\frac{1}{a} + \frac{1}{b}$
(c) $a - b$
(d) $a^2 - b^2$
(e) $a^3 + b^3$ (Hint: first compute $(a + b)(a^2 + b^2)$)

2. Let x_1, x_2 be roots of the equation $x^2 + 5x - 7 = 0$. Find
(a) $x_1^2 + x_2^2$
(b) $(x_1 - x_2)^2$
(c) $\frac{1}{x_2} + \frac{1}{x_2}$
(d) $x_1^2 + x_2^3$

3. Solve the following equations:
(a) $x^2 - 5x + 6 = 0$
(b) $x^2 = 1 + x$
(c) $\sqrt{2x + 1} = x$
(d) $x + \frac{1}{x} = 3$

4. Solve the equation $x^4 - 3x^2 + 2 = 0$

5. (a) Prove that for any $a > 0$, we have $a + \frac{1}{a} \geq 2$, with equality only when $a = 1$.
(b) Show that for any $a, b \geq 0$, one has $\frac{a+b}{2} \geq \sqrt{ab}$. (The left hand side is usually called the *arithmetic mean* of a, b; the right hand side is called the *geometric mean* of a, b.)