MATH 7: ALGEBRAIC EXPRESSIONS AND ROOTS

MATERIAL COVERED TODAY

Today we discussed how one works with algebraic expressions, i.e. expressions containing variables, such as $2(x + 1) - 3$. In particular, we discussed the following useful formulas:

1. $(ab)^n = a^n b^n$

2. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

3. $(a + b)^2 = a^2 + 2ab + b^2$

4. $(a - b)^2 = a^2 - 2ab + b^2$

5. $a^2 - b^2 = (a - b)(a + b)$

Replacing in the last equality a by $\sqrt{a}$, b by $\sqrt{b}$, we get

$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = a - b$$

which is very helpful in simplifying expressions with roots, for example:

$$\frac{1}{\sqrt{2} + 1} = \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

We also discussed solving simple equations: linear equation (i.e., equation of the form $ax + b = 0$, with a, b some numbers, and x the unknown) and equation where the left hand side is factored as product of linear factors, such as $(x - 2)(x + 3) = 0$.

HOMEWORK

1. Without a calculator, compute

$$199999 \cdot 200001$$

Is there a shorter way of doing it than the straightforward multiplication?

2. Simplify the following expressions, writing them in the form $\frac{f}{g}$, where f, g are polynomials.

(a) $\dfrac{1}{x + 1} - \dfrac{1}{x - 1}$ (b) $\left(1 + \dfrac{1}{x}\right) \div (x + 1)$ (c) $\left(1 + \dfrac{1}{x}\right) \div \left(1 - \dfrac{1}{x}\right)$

3. Factor (i.e., write as a product) the following expressions:

(a) $3x^3 - x^2 y + 6x^2 y - 2xy^2 + 3xy^2 - y^3$

(b) $a^2 - b^2 - 10b - 25$

(c) $x^2 + 4$

(d) $64 - a^8 b^8$

(e) $\frac{1}{9}x^2 - 25$

(f) $a^9 - 27$

(g) $(x - 2)^2 - (y + 3)^2$

(h) $4x^2 + 8xy + 4y^2$

(i) $(x - 2)^2 - 10(x - 1) + 25$

(j) $a^2 + 4ab + 4b^2$

(k) $a^2 - 2a + 1$

(l) $a^4 - b^4$ [Hint: $a^4 = (a^2)^2$.]

(m) $x^2 - 7$ [Hint: $7 = (\sqrt{7})^2$.]

4. Write each of the following expressions in the form $a + b\sqrt{3}$, with rational a, b:

(a) $(1 + \sqrt{3})^2$

(b) $(1 + \sqrt{3})^3$

(c) $\dfrac{1}{1 - 2\sqrt{3}}$

(d) $\dfrac{1 + \sqrt{3}}{1 - \sqrt{3}}$

(e) $\dfrac{1 + 2\sqrt{3}}{\sqrt{3}}$

5. Solve the equation $(x - 1)^2 = 6$.

6. Solve the following equations. Carefully write all the steps in your argument. Please do not use calculators.

(a) $(x^2 - 1)(x + 2) = 0$

(b) $\frac{x+2}{x+3} = 2$

(c) $5(x + 1) = 3x + 2$

(d) $(x - 3)(x + 4) = 0$

(e) $\frac{x^2 - 4}{x + 1} = x - 2$

(f) $x^2 + 4x = 0$

(g) $x^3 + 4x = 0$