

**MATH 7**  
**ASSIGNMENT 14: MORE COMBINATIONS...**

1. A club consisting of 30 members needs to choose a president, vice-president and treasurer. How many ways are there to do it?

**Answer:** we are choosing 3 persons out of 30, and the order matters (it is important who we choose to be the president and who the treasurer). So the answer is  ${}_{30}P_3 = 30 \cdot 29 \cdot 28$ .

You can also argue as follows: there are 30 ways to choose the president; after that, we have 29 ways to choose the vice-president; after that, 28 possible treasurers. Thus, the answer is  $30 \cdot 29 \cdot 28$ .

2. How many possible “hands” are there in poker?

**Answer:** A poker hand is a combination of 5 cards, chosen out of 52. The order doesn't matter: once you have 5 cards, it does not matter in which order you got them. So the answer is

$$\binom{52}{5} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960$$

3. How many “words” one can write using 5 letters A and 3 letters B (all 8 letters must be used)?

**Answer:** We have 8 positions and need to fill 5 of them with letters A; then the remaining 3 will be filled with B's. So we need to choose 5 items out of 8. Order doesn't matter: if we first put a letter A in 2nd position and then in 7th, the final result is the same as if we put first A in the 7th position and the second, in 2nd position. So the answer is  $\binom{8}{5}$ .

[Note: we could have also written the answer as  $\binom{8}{3}$  — it is the same number]

4. How many paths are there on the chessboard leading from bottom left to top right corner? [A path must always go to the right and up]

**Answer:** to get from bottom left to top right, we need to make 7 steps up and 7 steps to the right. Thus, every such path is described by a sequence of 14 steps, 7 of them up, 7 to the right, or by a “word” of 7 letters R and 7 letters U, written in some order. Thus, it is the same as in the previous example, only with different numbers, so we get  $\binom{14}{7}$ .

PROBLEMS

In all the problems, you can write your answer as a combination of factorials and  $\binom{n}{k}$ .

1. (a) How many 10-letter “words” one can write using 4 letters H and 6 letters T?  
(b) If we toss a coin 10 times and record the result as a sequence of letters H and T (writing H for heads and T for tails), how many different possible sequences we can get? How many of them will have exactly 6 tails?  
(c) If we toss a coin 10 times, what are the chances that there will be 6 tails? 3 tails? at least one tails?
2. If we randomly select 100 people from the population of the US, what are the chances that exactly 50 of them will be males? that at least 50 will be males? that all 100 will be males?
3. How many ways are there to divide 12 books
  - (a) Between two bags
  - \* (b) Between two bookshelves (order on each bookshelf matters!)
  - (c) Between three bags
  - \* (d) Between three bookshelves (order on each bookshelf matters!)
4. A person is running down the staircase. He is in a rush, so he may jump over some steps. If the staircase is 12 steps (including the top one, where he begins, and the last one, where he ends), in how many ways can he reach the bottom step in 5 jumps? What if there are no restrictions on the number of jumps? [Hint: keep track of the steps he steps on...]
5. [We talked about this problem before... Let's do it again!]
  - (a) For a group of 25 people, we ask each of them to choose a day of the year (non-leap, so there are 365 possible days). How many possible combinations can we get? [Order matters: it is important who had chosen which date]
  - (b) The same question, but now we additionally require that all chosen dates be different.
  - (c) In a group of 25 people, what are the chances that no two of them have birthday on the same day? that at least two have the same birthday?