

MATH 7
ASSIGNMENT 12: PASCAL TRIANGLE CONTINUED

PASCAL TRIANGLE

Recall the Pascal triangle:

				1								
				1	1							
				1	2	1						
				1	3	3	1					
				1	4	6	4	1				
				1	5	10	10	5	1			
				1	6	15	20	15	6	1		
				1	7	21	35	35	21	7	1	
				1	8	28	56	70	56	28	8	1

Every entry in this triangle is obtained as the sum of two entries above it. The k -th entry in n -th line is denoted by $\binom{n}{k}$, or by ${}_nC_k$. Note that both n and k are counted from 0, not from 1: for example, $\binom{2}{1} = 2$.

These numbers appear in many problems:

- The number of paths (going only right and up) from a square on the chessboard to the square m units up and k units to the right is ${}_{m+k}C_k$. For example, there ${}_{14}C_7$ paths from lower left to upper right of the usual 8×8 chessboard.
- The number of words of length exactly n which can be written using only 2 letters, say U and R, and which contain exactly k U's.
- and any more.. see problems below!

PROBLEMS

In this homework assignment (and in all other assignments in this class), many problems are non-trivial and require some thought. Try to start early. You are not expected to be able to solve all of the problems, so do not be discouraged if you can't solve some of them. The solutions are to be written on separate sheets of paper (as neatly as possible), with your name at the top, and handed back to me by the next class. Please make sure that you write not just the answer but also the solution, i.e. your reasoning showing how you arrived to this answer. Ideally, your solution should be such that someone who doesn't know how to solve this problem can read it and follow your arguments.

1. How many "words" of length 5 one can write using only letters U and R, namely 3 U's and 2 R's? What if you have 5 U's and 3 R's? [Hint: each such "word" can describe a path on the chessboard, U for up and R for right...]
2. How many sequences of 0 and 1 of length 10 are there? sequences of length 10 containing exactly 4 ones? exactly 6 ones?
3. If we toss a coin 10 times, what is the probability that all will be heads? that there will be exactly one tails? 2 tails? exactly 5 tails?
4. A drunkard is walking along a road from the pub to his house, which is located 1 mile north of the pub. Every step he makes can be either to the north, taking him closer to home, or to the south, back to the pub — and it is completely random: every step with can be north or south, with equal chances. What is the probability that after 10 steps, he will move
 - (a) 10 steps north
 - (b) 10 steps south
 - (c) 4 steps north

- (d) will come back to the starting position
5. If you have a group of 4 people, and you need to choose one one to go to a competition, how many ways are there to do it? if you need to choose 2? if you need to choose 3?
 - *6. How many ways are there to select 5 students from a group of 12?
 7. In a meeting of 25 people, each much shake hands with each other. How many handshakes are there altogether?
 8. If we draw 3 cards out of the deck, what are the chances that
 - They will all be all spades
 - They will be all aces
 - That they will be ace of spades, queen of spades, and king of spades, in this order
 - That they will be queen of spades, ace of spades, and king of spades, in this order
 - That they will be ace, queen, and king of spades, in some order