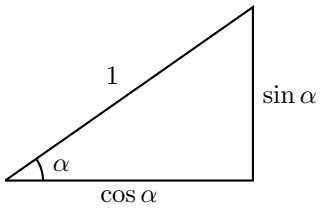
MATH 7: ASSIGNMENT 8

Basic trigonometry

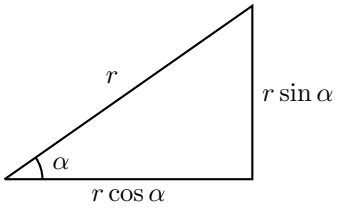
For any angle α, we define two numbers, $\sin\alpha$ (sine) and $\cos\alpha$ (cosine) as the lenghts of the legs in th right triangle with hypotenuse 1 and angle α:



In general, there is no simple formula for computing $\sin(\alpha)$ and $\cos(\alpha)$. However, there are some special angles, for which sin and cos can be computed explicitly:

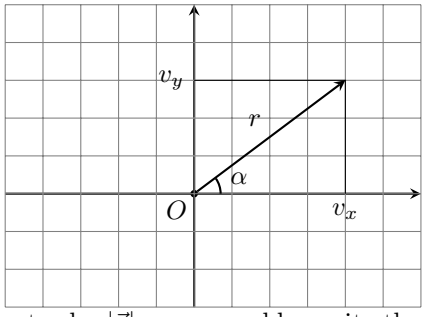
α	$\sin(\alpha)$	$\cos(\alpha)$
$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

Since any two right triangles with the same angles are similar, it shows that if we have a right triangle with angle α and hypotenuse r, then the sides will be $r\sin\alpha$ and $r\cos\alpha$:



In particular, this shows that if we have a vector $\vec{v}$ that has length r and forms angle α with the x axis, then its x and y components are

$$v_x = r\cos\alpha, \qquad v_y = r\sin\alpha$$



Note that it is also common to denote length of a vector by $|\vec{v}|$, so we could rewrite the previous formula by

$$v_x = |\vec{v}|\cos\alpha, \qquad v_y = |\vec{v}|\sin\alpha$$

Homework

In this homework, you can use the calculator to compute sin and cos of various angles.

1. Vector $\vec{v}$ has length 1; vector $\vec{w}$ has length 2, and the angle between them is 30°. What is the length of vector $\vec{v} + \vec{w}$? [Hint: introduce a coordinate system so that $\vec{v}$ goes along the x-axis, and write coordinates of each of the vectors in this system.]

2. A ship travels north for 3 miles , then turns and goes northeast for 2 miles, then north-northeast for another 5 miles. Where will it be at the end? how far east and north of the original position? [Northeast means that its direction bisects the angle between north and east directions, thus forming an angle of 45° with due north. North-northeast means that this direction bisects the angle between north and north-east, thus forming 22.5° angle with due north.]

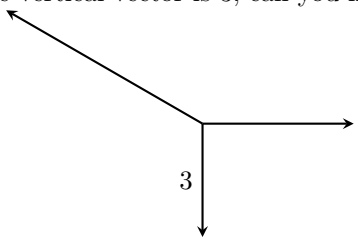
3. Consider a regular pentagon inscribed in a circle of radius 1. What is the side length of such a pentagon? [Hint: drop a perpendicular from the center to one of the sides and complete it to form a right triangle.]

4. Consider a parallelogram $ABCD$ with $AB = 1$, $AD = 3$, $\angle A = 40°$. Find the lengths of diagonals in this parallelogram.

5. Prove that the area of a triangle $\triangle ABC$ can be computed using the formula $A = \frac{1}{2} AB \cdot AC \cdot \sin \angle A$. [Hint: what is the altitude from vertex B?]

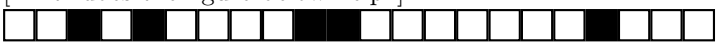
6. Find the area of a equilateral triangle with side 2.

7. In the figure below, one of the vectors is vertical, the other horizontal, and the third one forms 30° angle with the horizontal direction. The sum of all three vectors is zero. Knowing that the length of the vertical vector is 3, can you find the lengths of two other vectors?



***8.** Consider all numbers with at most six digits: 000001 – 999999. How many of them have the sum of digits equal to 17?

[Hint: does the figure below help?]



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