MATH 7
ASSIGNMENT 7: VECTORS

Summary of results

Here are is a summary of some geometric results we have proved using vectors:

Theorem. *In a parallelogram, diagonals bisect each other (i.e., the intersection point of the diagonals is the midpoint of each of them).*

Proof. See assignment 3, problem 5: if point A is at $(0,0)$, point B at (x_1, y_1), point D at (x_2, y_2), then the point M with coordinates $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ is the midpoint of BD and also midpoint of AC; thus, it is also the intersection point of the diagonals. □

Theorem. *The line connecting the midpoints of two sides of a triangle is parallel to the third side and in length, is exactly 1/2 of that side.*

Proof. See assignment 3, problem 7. □

Theorem. *Three medians of the triangle intersect at a single point. This point divides each of them in proportion 2:1.*

Proof. See assignment 6, problem 2. The intersection point is exactly the center of gravity (or center of masses) of the three vertices of the triangle, with the same mass at each of them. □

Problems

1. On each side of a parallelogram $ABCD$, mark a point which divides it in the proportion 2:1 (going clockwise). Prove that the marked points themselves form a parallelogram.
[Hint: denote $\overrightarrow{AB}=\vec{v}$, $\overrightarrow{AD}=\vec{w}$, and write vectors $\overrightarrow{AA_1}, \overrightarrow{AB_1}, \overrightarrow{A_1B_1}, \dots$ as combinations of $\vec{v}$, $\vec{w}$]

2. Consider the triangle $\triangle ABC$ with vertices at $(2,1)$, $(5,3)$, $(8,2)$. Find the coordinates of the intersection point of the medians.

3. Consider triangle ABC and let AA_1, BB_1, CC_1 be the medians of the triangle. Prove that then $\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = 0$.

4. In the notation of the previous problem, prove that the intersection point of the medians of $\triangle A_1B_1C_1$ is the same as intersection point of the medians of $\triangle ABC$. Can you find the ratio of areas $\frac{S_{A_1B_1C_1}}{S_{ABC}}$?

5. Consider a regular hexagon. If we place a unit mass at each vertex, where would be the center of mass of the resulting system? What about the regular pentagon?

6. Two airplanes are flying at the same altitude. At a given moment, airplane A is flying northwest at the speed of 300 mph and airplane B, which is 20 miles south from airplane A, is flying south at the speed of 200 mph.
(a) What is the velocity of airplane A? Write it as a vector, using coordinate system where y-axis points to the north. [You can use square roots in your answer.]
(b) Do the same for airplane B.
(c) What is the relative velocity of airplane B relative to airplane A?