MATH 7: ASSIGNMENT 4

RELATIVITY OF VELOCITY

The following rule is sometimes called Galileo relativity principle:

If point A is moving with velocity $\vec{v}_{rel}$ *relative to* B (i.e., observer at B sees A moving with this velocity) and B itself is moving with velocity $\vec{v}_B$, then the velocity of A is equal to $\vec{v}_A = \vec{v}_B + \vec{v}_{rel}$.

It can also be used to find the relative velocity: if A is moving with velocity $\vec{v}_A$, and B with velocity $\vec{v}_B$, then the velocity of A as seen by observer at B is equal to $\vec{v}_{rel} = \vec{v}_A - \vec{v}_B$.

CLASSWORK

1. If three vertices of a parallelogram have coordinates $(1, 2)$, $(3, 5)$ and $(2, 7)$, what are the coordinates of the fourth vertex? of the intersection point of the diagonals?

2. An observer drifting on an ice floe sees a boat, which is moving northwest at the speed of 15 mph (relative to the observer). However, the ice floe with the observer is itself drifting northeast at the speed of 2 mph (relative to the shore). Write the velocity of the boat relative to the shore as a vector (using a coordinate system where north is the positive direction of the y axis, as is usual on the maps).

HOMEWORK

1. Let $\vec{v} = (1, 1)$, $\vec{w} = (0, 2)$. Write each of the following vectors as a combination of $\vec{v}$ and $\vec{w}$, i.e. in form $c_1\vec{v} + c_2\vec{w}$, for some real numbers c_1, c_2.

(a) $(0, 1)$ (b) $(1, 0)$

(c) $(2, 1)$ (d) $(7, 5)$

2. (a) Let $A_1, A_2, \dots, A_6$ the the vertices of a regular hexagon inscribed in a circle of radius 1 with center a O. What is the sum $\overrightarrow{OA_1} + \overrightarrow{OA_2} + \dots + \overrightarrow{OA_6}$?

*(b) Can you answer the same question about a regular pentagon?

3. Consider the system of 5 points moving in the plane. At a given moment, the coordinates and velocities of these points are as shown below:

$A(2, 0)$; $\vec{v}_A = (4, 0)$
$B(3, 0)$; $\vec{v}_B = (6, 0)$
$C(0, -1)$; $\vec{v}_C = (0, -2)$
$D(-1, 1)$; $\vec{v}_D = (-2, 2)$
$E(1, 3)$; $\vec{v}_E = (2, 6)$

(as you can see, each point is moving in a straight line away from the origin, and the velocity is proportional to the distance from the origin).

What would the observer at point A see? what would be velocities of points B, C, D, E measured from A? Draw these velocity vectors on the plane; do you observe any pattern?

***4.** According to the widely accepted model of the universe, all stars are actually moving, with the velocity proportional to their position: if we introduce a coordinate system with the origin O at Sun, then each star S is moving away from the Sun at velocity $\vec{v}_S = c\,\overrightarrow{OS}$, where O is the current position of the Sun, S is the current position of the star S, and c is a certain constant, the same for all stars (Hubble constant).

Now if we put an observer on another star, say Alpha Centauri, what would he see? what will be the relation between the vector $\overrightarrow{AS}$ (where A is the current position of Alpha Centauri) and velocity of S relative to Alpha Centauri?

5. What is the probability that the sum of digits of a random two-digit number is 5?

6. Compute the product

$$(100 - 1^2)(100 - 2^2)\dots(100 - 25^2)$$