# MATH 6 — PREREQUISITES FOR MATH 7

## 1. Program

- Basics of logic. Knights and knaves. NOT, AND, OR, IF.
- Sets. Notation. Union, intersection, complement. Cardinality.
- Factorials and permutations.
- Ruler and compass constructions: midpoint, perpendicular, bisector.
- Coordinates. Equation of the line.
- Distance between two points on a coordinate plane. Equation of the circle.
- Arithmetic sequence. Geometric sequence. Formula for the general term. Formula for the sum.

## 2. Problems

**1.** On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "I and Sue are knights." So who is a knight and who is a knave?

**2.** On the island of Knights and Knaves, you meet three inhabitants:Bozo, Carl and Joe. Bozo says that Carl is a knave. Carl tells you, 'Of Joe and I, exactly one is a knight.' Joe claims, 'Bozo and I are different.'

**3.** On the island of Knights and Knaves, a traveler meets two inhabitants: Carl and Bill. Bill says: "Carl is a Knave". Carl says: "If Bill is a Knight, then I am a Knight, too."

**4.** Prove that

$$\text{NOT}(A \text{ AND } B) \text{ is the same as } (\text{NOT } A) \text{ OR} (\text{NOT } B)$$

**5.** Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)?
(a) $(A \text{ OR } B) \text{ AND} (A \text{ OR } C)$
(b) $A \text{ OR} (B \text{ AND } C)$.

**6.** If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,...

**7.** Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.
Denote:
$H$=set of all hearts cards
$Q$=set of all queens
$R$=set of all red cards
Describe by formulas (such as $H \cap Q$) the following sets:
all red queens
all black cards
all cards that are either hearts or a queen
all cards other than red queens
How many cards are there in each set?

**8.** Let
$A$=set of all people who know French
$B$=set of all people who know German
$C$=set of all people who know Russian

Describe in words the following sets:
(a) $A \cap B$ (b)$A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \overline{A}$.

**9.** In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?

**10.** Let $A = [1, 3] = \{x \mid 1 \leq x \leq 3\}$, $B = \{x \mid x \geq 2\}$, $C = \{x \mid x \leq 1.5\}$. Draw on the number line the following sets: $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.

**11.** Show that for two sets $A, B$, we have $|A \cup B| = |A| + |B| - |A \cap B|$.

**12.** A group of 6 club members always dine at the same round table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

**13.** In a computer game, a wizard is more powerful than an orc, so when a wizard fights an orc, he has 60% chance of winning. If a wizard fights one by one a group of 5 orcs, what are the chances that he will defeat them all?

**14.** In how many ways can one arrange 5 books on a shelf?

**15.** Show how to find a midpoint of an interval using ruler and compass.

**16.** Show how to construct a bisector using ruler and compass.

**17.** Draw all points on the plane for which one has $x = y + 1$.

**18.** Point $M$ has coordinates $(5, 7)$.
(a) Find coordinates of the point $M_1$ obtained from M by reflection around the $x$-axis
(b) Find coordinates of the point $M_2$ obtained from $M$ by reflection around the diagonal line.

**19.** Draw the graphs of the following functions:
(a) $2x + 3y = 1$
(b) $2x - 1 = y$
(c) $y = |x| - 2$

**20.** Find the distance between points $(2, 4)$ and $(3, 7)$.

**21.** Write the equation of the circle with center at $(1, 1)$ and radius 5.

**22.** What are the first 2 terms for the arithmetic sequence $a_1, a_2, 9, 2, 5, \ldots$ ?

**23.** In arithmetic sequence $a_{10} = 131$ and $d = 12$. What is $a_1$?

**24.** In arithmetic sequence $a_5 = 27$ and $a_{27} = 60$. Find the first term and the common difference.

**25.** Find the sum of the first 100 terms of the arithmetic sequence if $a_1 = 10$ and $a_{100} = 150$.

**26.** What are the first 2 terms for the geometric sequence $a_1, a_2, 24, 36, 54, \ldots$ ?

**27.** A geometric sequence has 99 terms, and the first term is 12 and the last term is 48. What is the 50th term?

**28.** Compute

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{10}}$$

**29.** Find the infinite sum

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots.$$