FACTORISATION: DIFFERENCE OF SQUARES, SUM/DIFFERENCE OF CUBES

1.Common Factor

The simplest type of factoring involves taking out a common factor from two or more terms.

$$14x^3y - 4x = 2x(7x^2y - 2).$$

2.Factoring in pairs

In some instances, there may be no common factor of all the terms in a given expression. It may, however, be useful to factor in pairs.

$$2a^3 + 3ab + 4a^2 + 6b = a(2a^2 + 3b) + 2(2a^2 + 3b) = (2a^2 + 3b)(a + 2)$$

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There are three special expansions and corresponding factorisations that frequently occur in algebra.

3. Factoring using the difference of squares

An identity is a statement in algebra that is true for all values of the unknowns.

By expanding, it is easy to show that $\boxed{(x - y)(x + y) = x^2 - y^2}$.

Examples:

Factorize

1. $x^2(x + 4) + 5(x + 4)$
2. $4x^2 + 16x + 2xy + 8y$
3. $x^2 - 2x - yx + 2y$
4. $100x^8y^2 - 16x^4y^6$
5. $x^4 - y^4$
6. $x^2 - 7$
7. Simplify $\frac{4x^2-25y^2}{2x-5y}$
8. Rationalize the denominator of $\frac{1}{\sqrt{5}-\sqrt{3}}$
9. Rationalize the denominator of $\frac{x}{x+\sqrt{y}}$

4. Factoring using the perfect squares

The other two basic algebraic identities are:

$\boxed{a^2 + 2ab + b^2 = (a + b)^2}$ and $\boxed{a^2 - 2ab + b^2 = (a - b)^2}$.

Factorize

1. $4x^2 - 12xy + 9y^2$
2. Simplify $\frac{x^2+5xy-11y^2}{x^2-16y^2}$ - $\frac{6xy}{2x(x-4y)}$

4. Factoring using the sum/difference of cubes

The difference of squares identity can be generalized to cubes. By expanding the right-hand side, we can show that $\boxed{a^3 - b^3 = (a-b)(a^2+ab+b^2)}$ and $\boxed{a^3 + b^3 = (a+b)(a^2-ab+b^2)}$.

Factorize

1. $(x+4)^3 - 9x - 36 =$
2. Rationalize the denominator of the fraction $\frac{2}{1+\sqrt[3]{4}}$
3. Rationalize the denominator of the fraction $\frac{2x}{\sqrt[3]{x}+\sqrt[3]{y}}$

5. Factoring using the sum/difference of powers of an odd $n = 2k+1$

$\boxed{a^n - b^n \text{ is a multiple of } (a-b)}$ and $\boxed{a^n + b^n \text{ is a multiple of } (a+b)}$.

Homework

1. Factorize
   (a) $3x^3 - x^2y + 6x^2y - 2xy^2 + 3xy^2 - y^3$
   (b) $a^2 - b^2 - 10b - 25$
   (c) $x^4 + 4$
   (d) $x^4 + 64$
   (e) $64 - a^8b^8$
   (f) $a^4 - 100$
   (g) $\frac{1}{9}x^2 - 25$
   (h) $a^9 - 27$
   (i) $(x-2)^2 - (y+3)^2$
   (j) $4x^2 + 8xy + 4y^2$
   (k) $4x^2 + 12xy + 9y^2$
   (l) $(x-2)^2 - 10(x-1) + 25$
   (m) $t^3 - t^2 + t - 1$
   (n) $t^3 - t^2 - t + 1$
   (o) Rationalize the denominator of $\frac{4}{\sqrt{2}+\sqrt{5}}$
   (p) Rationalize the denominator of $\frac{x^2y}{x-\sqrt{y}}$
   (q) Rationalize the denominator of the fraction $\frac{1}{a-\sqrt[3]{b}}$
2. The real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x+y$?
3.* The number $(2^{48} - 1)$ is exactly divisible by two numbers between 60 and 70. Find the numbers.
4.* Is the number
$$x = 2222^{5555} + 5555^{2222} = (2222^5)^{1111} + (5555^2)^{1111}$$
divisible by 7?
5.* Use the difference of squares and difference/sum of cubes to find the greatest power of 2 that is a factor $10^{1002} - 4^{501}$