Homework 9
Factoring and Quadratic Equation Continued

Math 7a

November 27, 2017

In this homework, please do not make use of a calculator or a computer and make sure you show your work! In class, we derived a convenient way of solving a quadratic equation of form:

$$ax^2 + bx + c = 0$$

We can proceed by "completing the square". For example:

$$x^2 + 6x + 2 = x^2 + 2 \cdot 3x + 9 - 7 = (x+3)^2 - 7 = (x + 3 - \sqrt{7})(x + 3 + \sqrt{7})$$

Therefore, we obtain: $x + 3 - \sqrt{7} = 0$ or $x + 3 + \sqrt{7} = 0$, which gives $x = -3 + \sqrt{7}$, or $x = -3 - \sqrt{7}$. Or in general, assuming we divided the equation by a constant to get $a = 1$:

$$\begin{aligned} x^2 + bx + c &= x^2 + 2\frac{b}{2}x + c \\ &= \big(x^2 + 2\frac{b}{2}x + \big(\frac{b}{2}\big)^2\big)^2 - \big(\frac{b}{2}\big)^2 + c \\ &= \big(x + \frac{b}{2}\big)^2 - \frac{b^2 - 4c}{4} \\ &= \big(x + \frac{b}{2} - \sqrt{\frac{b^2-4c}{4}}\big)\big(x + \frac{b}{2} + \sqrt{\frac{b^2-4c}{4}}\big) \\ &= \big(x + \frac{b - \sqrt{b^2-4c}}{2}\big)\big(x + \frac{b + \sqrt{b^2-4c}}{2}\big) \end{aligned} \tag{1}$$

This makes sure our solutions are:

$$x = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

Or in case $a = 1$ isn't true:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We also derived the Vieta formula for polynomials of degree 3: if a cubic polynomial of form:

$$ax^3 + bx^2 + \ldots c$$

has 3 roots x_1, x_2, and x_3: then we can rewrite it as:

$$ax^3 + bx^2 + \ldots c = a(x - x_1)(x - x_2)(x - x_3)$$

Then expanding the right-hand side, we see that:

$$x_1 + x_2 + x_3 = -\frac{b}{a}$$

$$x_1 \cdot x_2 \cdot x_3 = -\frac{c}{a}$$

1. Solve the following quadratic equations using whichever method you prefer:
   (a) $x^2 - 14x + 3 = 0$
   (b) $2x^2 - 14x + 9 = 0$
   (c) $3x^2 - 33x + 3 = 0$
   (d) $x^2 - 12x - 3 = 0$
   (e) $x^2 + 2x - 9 = 0$
2. Solve the following cubic equations:
   (a) $x^3 + 7x^2 + 15x + 9 = 0$
   (b) $x^3 - 7x^2 + 14x - 8 = 0$
   (c) $x^3 - 11x^2 + 19x - 9 = 0$
   (d) $2x^3 - 12x^2 + 24x - 16 = 0$
   (e) $3x^3 - 15x^2 - 51x + 63 = 0$
3. For what values of "b" has the function $x^2 + bx + 14$
   (a) no roots?
   (b) exactly one root?
   (c) 2 distinct roots?

   Hint: how many square roots do numbers have?