# Classwork 6
# Trigonometry Continued: $tan(\alpha)$ and Trigonometric Identities

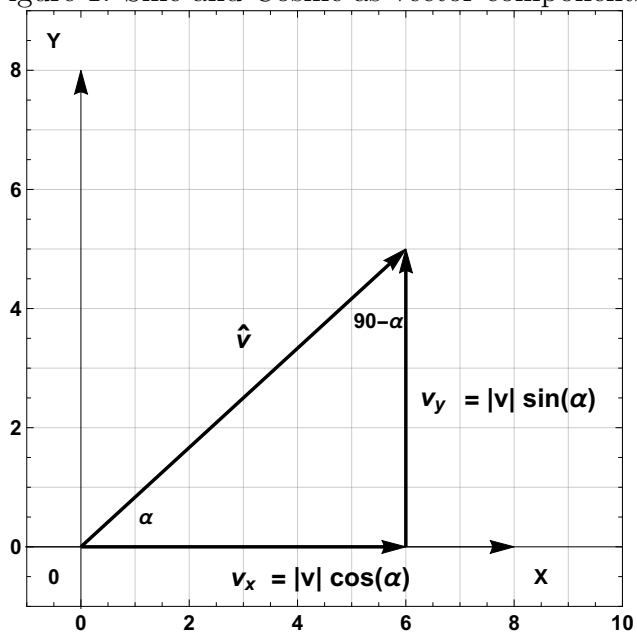
Math 7a

October 28, 2017

## 1 Review Homework 5

## 2 Vector Components

Figure 1: Sine and Cosine as vector components.




Remembering from last time:

$$sin(\alpha) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{5}{\sqrt{5^2 + 6^2}}$$

$$cos(\alpha) = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{6}{\sqrt{5^2 + 6^2}}$$

| Trigonometric Functions | | | | | | |
|---|---|---|---|---|---|---|
| Function | Notation | Definition | 0 | 30 | 45 | 60 |
| Sine | $sin(\alpha)$ | $\frac{\text{opposite side}}{\text{hypotenuse}}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| Cosine | $cos(\alpha)$ | $\frac{\text{adjacent side}}{\text{hypotenuse}}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ |

## 2.1 Problems

1. A high-schooler bikes 5 km north and then turns right and bikes for another 12 km to the east. What is the displacement of the high-schooler from the original location?

2. The same high-schooler now bikes 5 km north, turns right and bikes for 24 km east, after which turns left and bikes for an additional 2 km. What is the displacement of the high-schooler from the original location now?

3. If vector $\overrightarrow{v}$ has magnitude 2 and vector $\overrightarrow{u}$ has magnitude 3, and the angle between them is 30 degrees, what is the magnitude of $\overrightarrow{v} + \overrightarrow{u}$?

# 3 Tangent $tan(\alpha)$

Now we can also define the 3rd trigonometric ratio (see Figure 2):

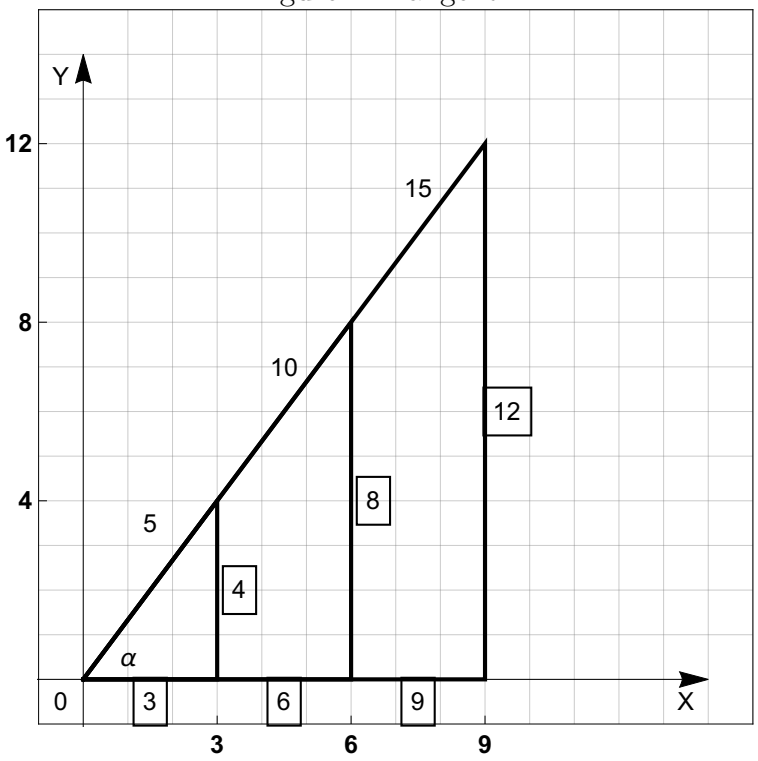
$$tan(\alpha) = \frac{sin(\alpha)}{cos(\alpha)} = \frac{\text{opposite side/hypotenuse}}{\text{adjacent side/hypotenuse}} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{4}{3} = \frac{8}{6} = \frac{12}{9}$$

| Trigonometric Functions | | | | | | |
|---|---|---|---|---|---|---|
| Function | Notation | Definition | 0 | 30 | 45 | 60 |
| Sine | $sin(\alpha)$ | $\frac{\text{opposite side}}{\text{hypotenuse}}$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| Cosine | $cos(\alpha)$ | $\frac{\text{adjacent side}}{\text{hypotenuse}}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ |
| Tangent | $tan(\alpha)$ | $\frac{\text{opposite side}}{\text{adjacent side}}$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |

## 3.1 Problems

1. If a right triangle $\Delta ABC$ has sides $AB = 3 * \sqrt{3}$ and $BC = 9$, and side $AC$ is the hypotenuse, find all 3 angles of the triangle.

Figure 2: Tangent.




2. A high-schooler bikes 10 km north and then makes a right turn and bikes for $x$ km to the east. If the displacement vector makes 30 degree angle with the north, what is $x$? That is how many km did our high-schooler bike to the east?

3. Let vectors $\overrightarrow{v} = (3-\sqrt{3}, \sqrt{3}/2)$ and $\overrightarrow{u} = (\sqrt{3}-2, \sqrt{3}/2)$. Find the angle vector $\overrightarrow{v} + \overrightarrow{u}$ makes with the $x$-axis.

# 4 Trigonometric Identities and Laws of Sines and Cosines

The most prominent trigonometric identity is given as:

$$\sin^2(\alpha) + \cos^2(\alpha) = 1.$$

Let us try to derive it:

1. A right triangle with hypotenuse $c$ and an angle $\alpha$ is given. Express the remaining 2 sides ($a$ and $b$) of triangle using only $c$ and $\alpha$.

2. Using expressions obtained for $a$ and $b$, express the hypotenuse $c$ and simplify.

**Law of Sines:** Given a triangle $\Delta ABC$ with sides $a$, $b$, and $c$ (see Figure 3), the following is always true:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

Figure 3: Law of Sines