MATH 6: ASSIGNMENT 9

Factorials and permutations

If we are choosing k objects from a collection of n so that a)order matters and b)no repetitions allowed, then there are

$$n(n-1)\ldots \quad (k \text{ factors})$$

ways to do it.

In particular, if we take $k = n$, it means that we are selecting one by one all n objects — so this gives the number of possible ways to order n objects:

$$n! = n(n-1)\cdot\dots\cdot 2\cdot 1$$

(reads n factorial).

For example: there are $52!$ ways to mix the cards in the usual card deck.

Note that the number $n!$ grow very fast: $2! = 2$, $3! = 6$, $4! = 2\cdot 3\cdot 4 = 24$, $5! = 120$, $6! = 620$

In all the problems that ask you to compute something, it suffices to write an expression for the answer, e.g., $1/2^{11}$; it is not necessary to actually perform the multiplication.

1. (a) In a class of 25 students, everyone chooses a date (e.g., March 13). How many combinations are possible? (Students only choose month and day, not year; February 29th is not allowed, so there are 365 different possibilities. Also, it matters who had chosen which day: combination where Bill has chosen March 12 and John, June 15 is considered different from the one where Bill has chosen June 15 and John March 12.)

(b) In the same situation, how many such combinations are possible if we additionally require that all dates must be different?

*(c) Suppose now that each of these 25 students has chosen a date at random, not knowing the choices of others. What is the probability that all of these dates will be different? That at least 2 will coincide?

2. About 1/6 of Americans have blue eyes. If we choose 10 people at random, what is the probability that all of them have blue eyes? that none has blue eyes? that at least one has blue eyes?

3. A group of 6 club members always dine at the same table in the club; there are exactly 6 chairs at the table. They decided that each day, they want to seat in a different order. Can they keep this for a year? Two years?

4. How many ways are there to seat 15 students in a classroom which has 15 chairs? If the room has 25 chairs?

5. A puzzle consists of 9 small square pieces which must be put together to form a 3×3 square so that the pattern matches (this kind of puzzles is actually quite hard to solve!). It is known that there is only one correct solution. If you started trying all possible combinations at random, doing one new combination a second, how long will it take you to try them all?

6. 10 people must form a circle for some dance. In how many ways can they do this?

7. At a fair, they offer you to play the following game: you are tossing small balls in a large crate full of empty bottles; if at least one of the balls lands inside a bottle, you win a stuffed toy (worth about $5). Unfortunately, it is really impossible to aim, so the game is just a matter of luck (or probability theory): every ball you toss has a 20% probability of landing inside the bottle.

(a) If you are given three balls, what is the probability that all three will be hits? That all three will be misses? That at least one will be a hit?

(b) Same questions for five balls.

*(c) They charge you 2 dollars for 3 balls, or 3 dollars for 5 balls. Which is a better deal? [Considering only from the point of view of the chances of winning, not the fun you are getting]