MATH 6
ASSIGNMENT 6: SETS

Sets

By word *set*, we mean any collection of objects: numbers, letters,.... Most of the sets we will consider will consist either of numbers or points in the plane. Objects of the set are usually referred to as *elements* of this set.

Sets are usually described in one of two ways:

- By explicitly listing all elements of the set. In this case, curly brackets are used, e.g. $\{1, 2, 3\}$.
- By giving some conditions, e.g. "set of all numbers satisfying equation $x^2 > 2$". In this case, the following notation is used: $\{x \mid \dots\}$, where dots stand for some condition (equation, inequality, ...) involving x, denotes the set of all x satisfying this condition. For example, $\{x \mid x^2 > 2\}$ means "set of all x such that $x^2 > 2$".

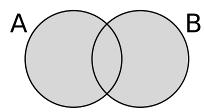
Other notation:

$x \in A$ means "x is in A", or "x is an element of A"

$x \notin A$ means "x is not in A"

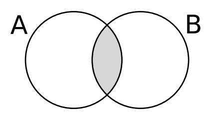
$A \cup B$: union of A and B. It consists of all elements which are in either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}.$$



$A \cap B$: intersection of A and B. It consists of all elements which are in both A and B:

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}.$$



$\overline{A}$: complement of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.

Homework

1. Consider the operation NOR which is just the opposite of OR: it returns T only if both A and B are F. Using only the component NOR, see if you can create circuits equivalent to AND, OR, and NOT as you did in the previous problem.
2. Using only AND, NOT, and OR, produce a three-input AND circuit, i.e., the output is F unless all three inputs are a T. (You do not have to use all three circuit elements.)
3. Using only AND, NOT, and OR, produce a three-input OR circuit, i.e., the output is T if any of the inputs is T.
4. If Al comes to a party, Betsy will not come. Al never comes to a party where Charley comes. And either Betsy or Charley (or both) will certainly come to the party.
   Based on all of this, can you explain why it is impossible that Al comes to the party?
5. Let
   A=set of all people who know French
   B=set of all people who know German
   C=set of all people who know Russian
   Describe in words the following sets:
   (a) $A \cap B$ (b) $A \cup (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $C \cap \overline{A}$.
6. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.
   Denote:
   H=set of all hearts cards
   Q=set of all queens
   R=set of all red cards
   Describe by formulas (such as $H \cap Q$) the following sets:
   all red queens
   all black cards
   all cards that are either hearts or a queen
   all cards other than red queens
   How many cards are there in each set?
7. In a class of 25 students, 10 students know French, 5 students know Russian, and 12 know neither. How many students know both Russian and French?
8. Draw the following sets on the number line:
   (a) Set of all numbers x satisfying $x \leq 2$ and $x \geq -5$;
   (b) Set of all numbers x satisfying $x \leq 2$ or $x \geq -5$
   (c) Set of all numbers x satisfying $x \leq -5$ or $x \geq 2$
9. For each of the sets below, draw it on the number line and then describe its complement:
   (a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c) $(0, 5) \cup (2, \infty)$ where
   $[a, b] = \{x \mid a \leq x \leq b\}$ is the interval from a to b (including endpoints),
   $(a, b) = \{x \mid a < x < b\}$ is the interval from a to b (**not** including endpoints),
   $[a, \infty) = \{x \mid a \leq x\}$ is the half-line from a to infinity (including a),
   $(a, \infty) = \{x \mid a < x\}$ is the half-line from a to infinity (**not** including a)