

SchoolNova, Math 5c
Homework 21
Warm up for Math Battle!
May 13, 2018

1. Consider the sequence $7, 7^2, 7^3, \dots$. Show that there are two numbers in this sequence which have the same last two digits.
2. A deck of cards is dealt out.
 - (a) What is the probability that the fourth card is an ace?
 - (b) What is the probability that the first ace occurs on the fourth card?
3.
 - (a) An urn contains 5 red and 7 green balls. We draw one ball from the urn. What is the probability that it is red?
 - (b) We put back the previously drawn ball.
 - (c) Next, we draw 2 balls **without replacement**. What is the probability that both balls are red?
4.
 - (a) Roll two dice. What is the probability that both show the same face?
 - (b) Roll three dice. What is the probability that exactly two of them show the same face? What is the probability that at least two of them show the same face?
 - (c) Roll four dice. What is the probability that **no two** show the same face (that is, all faces are different)?

5. Solve the equation

$$1 - 1999(1 - 1999(1 - 1999x)) = x.$$

6. Find the sum:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} \cdots \frac{1}{11 \times 12}$$

(There is a clever trick)

7. Given

$$S = 1! + 2! + 3! + \dots + 98! + 99!$$

then the units digit in the value of S is

- (a) 9 (b) 8 (c) 5 (d) 3 (e) 0

8. Given three numbers, x , x^x and x^{x^x} , where $0.9 < x < 1.0$, arrange the numbers in order of increasing magnitude.
9. Show that if n and m are divisible by d , then each of the following numbers is divisible by d :
 (a) $n + m$ (b) $5n + 3m$
10. The Fibonacci sequence is given by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$, for $n > 2$. Here are the first few Fibonacci numbers:

1, 1, 2, 3, 5, 8, ...

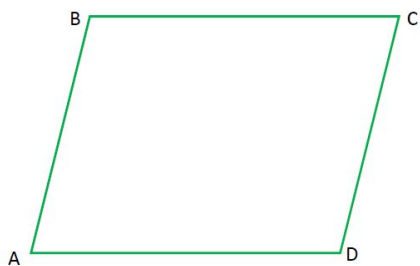
- (a) List the first 20 Fibonacci numbers.
 (b) Show that

$$\binom{3}{0} + \binom{2}{1} = F_4. \tag{1}$$

- (c) Show that

$$\binom{4}{0} + \binom{3}{1} + \binom{2}{2} = F_5. \tag{2}$$

11. A parallelogram is a quadrilateral, in which opposite sides are parallel.



- (a) Show that in a parallelogram, diagonally opposite angles are equal, that is, $\angle A = \angle C$ and $\angle B = \angle D$.
- (b) Show that opposite sides are equal, that is $\overline{AB} = \overline{CD}$, and $\overline{BC} = \overline{AD}$. (Hint: Use the properties of triangles)