

SchoolNova, Math 5c
Homework 16
Basic Probability Theory
March 4, 2018

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a *. If unable to solve a problem, please present your thoughts and any partial solution.

1. (a) A fair die is rolled. What is the probability of an even outcome?
(b) A fair die is rolled twice. What is the probability of an even outcome on both rolls?
2. A fair die is rolled twice. What is the probability that the sum of the two rolls is 7?
3. A fair die is rolled twice. What is the probability that the number on the first roll is less than the number on the second roll?
4. (a) We have a fair coin. We toss it three times. What is the probability that all tosses show the same face (that is, three heads or three tails)?
(b) We have a coin that is **not fair**. Now, probability of heads is $2/3$ and probability of tails is $1/3$. We toss it three times. What is the probability that all tosses show the same face?
- 5.* A deck of cards is dealt out.
 - (a) What is the probability that the fourth card is an ace?
 - (b) What is the probability that the first ace occurs on the fourth card?
6. (a) An urn contains 5 red and 7 green balls. We draw one ball from the urn. What is the probability that it is red?
(b) We put back the previously drawn ball.
(c)* Next, we draw 2 balls **without replacement**. What is the probability that both balls are red?
- 7.* In how many ways can you place 4 rooks on a 4×4 chessboard, so that no rook can capture any other? That is, no row or column contains more than one rook.
- 8.* Consider the following equation

$$x_1 + x_2 + \dots + x_r = n, \tag{1}$$

where r and n are positive integers and $r \leq n$. Find the **number** of solutions of the above equation, such that x_1, x_2, \dots, x_r are all integers, and each of $x_1 > 0, x_2 > 0, \dots, x_r > 0$. The answer should be in terms of r and n . Hint: Work out some examples, say for $n = 4$ and various values of r ; and $n = 5$ and various values of r .