

SchoolNova, Math 5c
Homework 14
Permutations and Combinations
February 4, 2018

A Combinatorics

- I. There are $n!$ **permutations**, that is, ordered arrangements of n objects.

For example, given 3 balls, **R**ed, **B**lue and **G**reen, we **list** below all the permutations of these balls:

R B G

R G B

B R G

B G R

G R B

G B R

So, there are $3! = 6$ such permutations.

- II. There ${}^nC_r = \frac{n!}{r!(n-r)!}$ **combinations**, that is, number of different ways of selecting r objects out of n .

- (a) A woman has 4 friends, **R**ebecca, **P**riyanka, **C**aroline and **J**ennifer. In how many ways can she invite 2 out of the 4 friends to a tea party?

We list here all the different combinations of 2 out of the 4 friends:

R P P C

R C P J

R J C J

With $n = 4$ and $r = 2$, there are ${}^4C_2 = \frac{4!}{2!2!} = 6$ ways of doing so.

- (b) If two of her friends Caroline and Jennifer are feuding, in how many ways can she now invite 2 out of the 4 friends?

We now exclude the case C J from the list in (a). Therefore, there are a total of 5 such ways, as listed below:

R P P C

R C P J

R J

- III. A **partitioning** of n objects into 3 distinct groups of sizes r_1 , r_2 and r_3 , with $r_1 + r_2 + r_3 = n$ can be done in $\frac{n!}{r_1!r_2!r_3!}$ different ways.

B Assignment

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a *. If unable to solve a problem, please present your thoughts and any partial solution.

1. You are given 4 balls: **Red**, **Red**, **Blue** and **Green**. The two red balls are alike. List all the permutations of these balls:
2. How many 3 digit odd numbers are there?
3. Pascal has 5 books, of which 3 are Mathematics and 2 are Physics. He arranges his books on the bookshelf such that all the books in the same subject are together. How many different arrangements are possible?
4. (a) The Mathematics department in a school has 4 mathematics teachers, **Diophantus**, **Euclid**, **Hypatia**, and **Plato**. 2 of these teachers will be teaching Geometry, 1 of them will be teaching Algebra and 1 will be teaching Probability. List all the different ways this can be done.
(b)* If there are a total of 7 mathematics teachers in the department, and 3 of these teachers are to teach Geometry, 2 are to teach Algebra and 2 are to teach Probability, in how many ways can this be done? Use the expressions given in A.
5. (a) 4 players P_1 , P_2 , P_3 and P_4 are to be divided into 2 teams of 2 each. Call the teams Team 1 and Team 2. List all the different ways this can be done.
(b)* 6 players are to be divided into 2 teams of 3 each. Call the teams Team 1 and Team 2. In how many ways can this be done? Use the expressions given in A.