SchoolNova, Math 5c
Homework 13
Algebraic Expressions and Factorization
January 28, 2018

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a $*$. If unable to solve a problem, please present your thoughts and any partial solution.

1. Factorize the following algebraic expressions:
   (a) $10x - 15x^2$
   (b) $11a^2b + 33ab^2$
   (c) $25a^2 - 49b^2$
   (d) $x^4 - y^4$
   (e) $3a^4 - 48b^4$

2. Factorize the following algebraic expressions:
   (a) $11(x+3) + 4(x+3)$
   (b) $3(x-2y) + 7(x-2y)$
   (c) $r(y-x) + s(x-y)$
   (d) $5(x-2y)^2 + 3(x-2y)$

3. Factorize the following algebraic expressions:
   (a) $ax + bx + ay + by$
   (b) $ax^2 + by^2 + bx^2 + ay^2$
   (c) $a^2 + bc + ab + ac$
   (d) $a^2 + 2a + ab + 2b$
   (e) $x^2 - xz + xy - yz$

4. Simplify the following expressions:
   (a)
   $$\frac{1}{x+1} - \frac{1}{x-1}$$
   (b)
   $$\left(1 + \frac{1}{x}\right) \div (x+1)$$
   (c)
   $$\left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$

5. $*$ Write each of the following expressions in the form $a + b\sqrt{3}$, with rational a and b:

   (a) $(1 + \sqrt{3})^2$

   (b) $(1 + \sqrt{3})^3$

   (c) $\frac{1}{1-2\sqrt{3}}$

   (d) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

6. The number 100 is multiplied either by 2 or by 3, then the result is increased either by 1 or by 2, and then the new result is divided either by 3 or by 4. If the final result is a natural number, what is the final result?

7. Carla wants to fold a cube from a paper net. By mistake, she drew 7 squares on her sheet instead of 6 squares. Which square(s) can she remove so that the figure remains connected, and Carla can fold a cube from it?