SchoolNova, Math 5c
Homework 8
Pigeonhole Principle
December 3, 2017

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a $*$. If unable to solve a problem, please present your thoughts and any partial solution. The first four problems are based on **Pigeonhole Principle**.

1. Show that among the years $2011, 2012, \ldots 2019$, there will be two in which January 1 falls on the same day of the week.

2. A fifth-grader named Saaketh holds all his socks in the same drawer: 20 black socks, 12 white socks, and 4 green socks.

   (a) How many socks does he have to get from the drawer, without looking, to be certain to get at least one matching pair?

   (b) How many socks does he have to get from the drawer, without looking, to be certain to get at least three matching pairs, all of the same color?

3. (a) Consider all numbers which are written only using the digit 1: $1, 11, 111, 1111, \ldots$. Show that there are two such numbers which give the same remainder upon division by 31.

   (b) Find two such numbers which give the same remainder, using the modulus calculator: $https://www.miniwebtool.com/modulo-calculator/$

4. $*$ Consider the sequence $7, 7^2, 7^3 \ldots$. Show that there are two numbers in this sequence which have the same last two digits.

5. The average of five numbers is 6. If one of the five numbers is removed, the average of the remaining four numbers is 7. What is the value of the number that was removed?

6. In a 100 meter race, Marc beat Peter by 5 meters: at the moment Marc finished, Peter was 5 meters behind. For the next race, to make the chances even, the coach moved Marc's start back 5 meters so he has to run 105 meters. Who will win: Marc or Peter?

7. Michael and Alex are playing the following game: Michael thinks of a number between 1 and 100; Alex tries to find the number by asking questions which can be answered yes/no. However, Michael is allowed to lie once. What is the fastest way to find out this number? How many questions do you need?

8. In the multiplication example given here, each of A, B and C stands for a different digit and each of the blank spaces represents a non-zero digit. What digits do A, B, and C each represent?

$$\begin{array}{cccccc} & & & A & B & C \\ & & \times & A & B & C \\ \hline & & \star & \star & \star & 9 \\ & \star & \star & \star & 4 & \\ \star & \star & \star & 1 & & \\ \hline \end{array}$$

9. Given below is the sum of two three-digit numbers. A, B and C represent the digits 2, 3 and 5, but not necessarily in the same order, and different letters represent different digits. What is the largest value the indicated sum could have?

$$\begin{array}{cccc} & B & A & C \\ + & C & A & B \\ \hline \end{array}$$