

SchoolNova, Math 5c
Homework 7
Algebraic Equations with Absolute Values and Inequalities
November 5, 2017

Please provide sufficient details about how you solved the problem. More difficult problems are marked with a *. If unable to solve a problem, please present your thoughts and any partial solution.

1. Show that each of the following numbers are solutions to the given equations or inequalities:

- (a) $x = 3$ in $x^2 - 9 = 0$.
- (b) $x = -5$ in $x^2 \geq 16$.
- (c) $x = -1$ in $2x - 3 \leq -2$.
- (d) $x = 1$ in $2(x - 5) \leq 4x$.
- (e) $x = 5$ in $|4 - x| \leq 2$.

2. Solve the following equations containing absolute values, and plot on a number line:

- (a) $|x| = 3$.
- (b) $|x + 2| = 7$.
- (c) $|2x - 5| = 9$.
- (d) $|3x - 12| = 0$.
- (e) $|2x - 3| - 4 = 3$.
- (f) $|4x + 3| = 3 - x$.
- (g) $|2x - 12| = |4x|$.

3. Solve and or plot the following inequalities on a number line:

- (a) $x > -1$ and $x < 2$.
- (b) $x \leq -4$ and $x \geq 4$.
- (c) $x^2 \leq 9$.
- (d) $-x < -5$.
- (e) $|x - 5| \leq 3$.

4. Evaluate the following algebraic expressions for $x = 3$ and $y = 7$:

- (a) $2x + 3$
- (b) $x^2 + y^2$
- (c) $(x + y)^2$
- (d) $-x^2 - y^2 + 3$
- (e) $3x^3y^2$

5. Simplify each of the following algebraic expressions, by opening the parenthesis and collecting like terms:
- (a) $a(a + b) + b(a + b)$
 - (b) $(a + b)(a + b)$
 - (c) $a(a + b) - b(a + b)$
 - (d) $(a - b)(a + b)$
 - (e) $(a + b)^2$
 - (f) $(a + b)^3$
 - (g) $(a + b + c)^2$
6. A group of soldiers need to cross a river. It is too fast to swim across. Fortunately, they see two boys fishing from a small boat who are ready to help. However, the boat is so small that it can only hold one soldier or two boys. Is there a way for all of the soldiers to get to the other side and return the boat to the boys?
7. (a) An ATM machine dispenses cash using \$20 and \$50 bills. What sum can one withdraw using this ATM?
- (b) An ATM machine in the imaginary country of Khiva dispenses cash using 15 tugrik and 35 tugrik (tugrik is the name of the local currency). What sums can one withdraw using this ATM?
8. * Show that $n^5 - n$ is divisible by 5, where n is an integer.