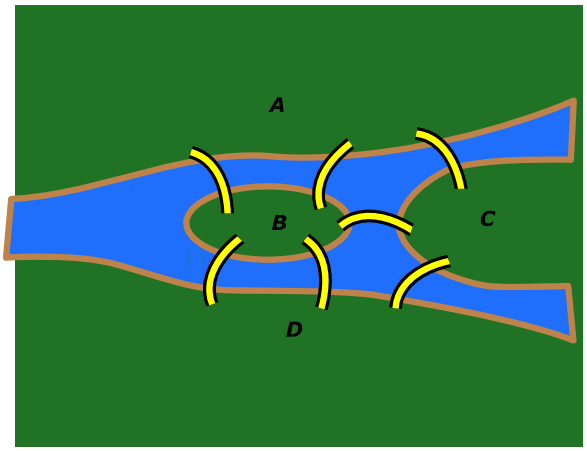
MATH 5: ASSIGNMENT 11

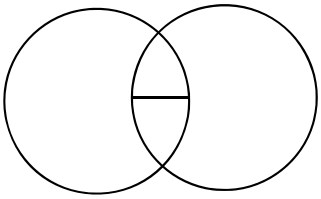
DECEMBER 17, 2017

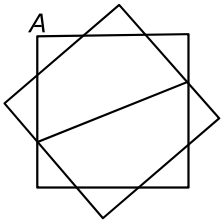
Lets talk about the city of Königsberg. Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands, which were connected to each other and the mainland by seven bridges. The problem was to find a walk through the city that would cross each bridge once and only once. The islands could not be reached by any route other than the bridges, and every bridge must have been crossed completely every time; one could not walk halfway onto the bridge and then turn around and later cross the other half from the other side. The walk need not start and end at the same spot.

1. The figure to the right shows a map of a city with islands and bridges. Is it possible to complete a walk in this city so that you walk on each of the seven bridges exactly once? (You may start anywhere you like, and you do not have to come back to the starting point.)



2. For the following figures, is it possible to draw them starting at point A, without lifting your pencil off the paper and drawing each line exactly once, without retracing? If not, can you do it starting at some other point?





What Euler and we found: the key to answering this kind of problems lies in counting the number of islands with odd number of bridges — or, if you replace each island by a point and each bridge by an arc, the number of points which have an odd number of arcs coming into them. If there are at most two such points, the problem can be solved; if more than two, it cannot be solved.

Movie about Konigsberg, Kalingrad, and bridge problem we watched in class
http://www.youtube.com/watch?v= OiZrmnni9Y

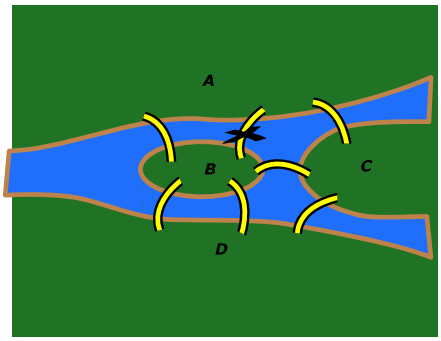
Euler's biography :
http://www.youtube.com/watch?v=Ty6ejK1rAkg

More Euler's biography, good biography, but read by tomb voice narrator:
http://www.youtube.com/watch?v=ZFG9LJXFtX0

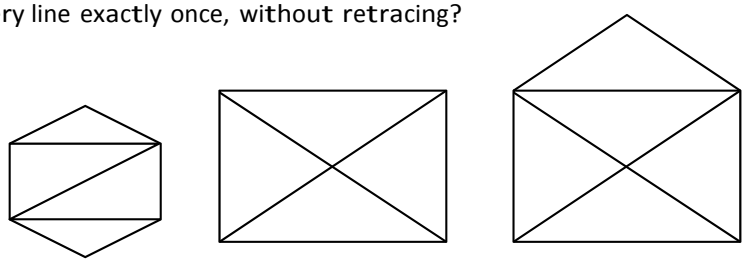
Lot's of info in a fun style
http://www.youtube.com/watch?v=R5a4CVKTZek

Homework

1. Once, flood destroyed one of the bridges between riverbank A and island B; the new map (with the destroyed bridge crossed out) is shown here. Is it now possible to complete a walk in this city so that you walk on each of the seven bridges exactly once?

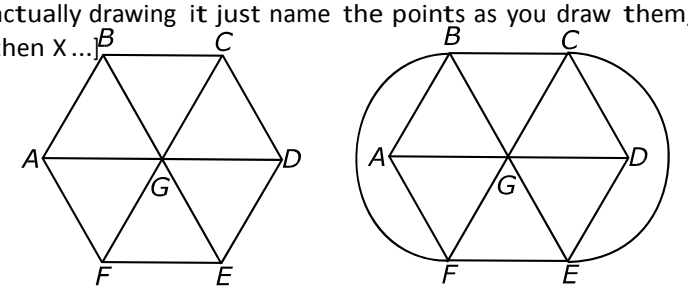


2. Which of the figures below can be drawn without lifting your pencil from the paper and drawing very line exactly once, without retracing?

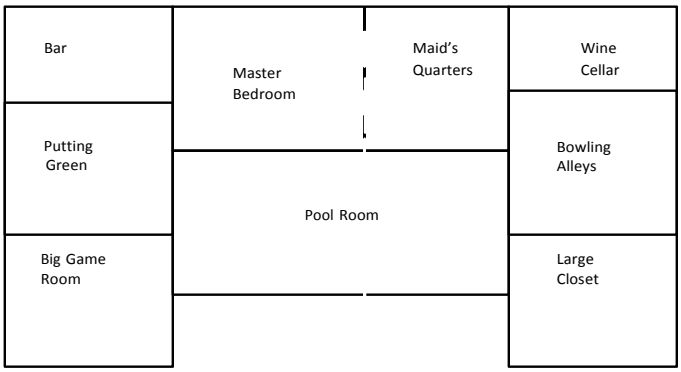


[Hint: it is easier if you start at one of the points where 3 lines meet.]

3. Can you draw the following figures without lifting your pencil off the paper and drawing each line exactly once, without retracing? [To make it possible for me to read your solution, instead of actually drawing it just name the points as you draw them, e.g. "Start at A, then go to B, then C, then X ...]



4. The figure below shows a plan of a house.



Is it possible to enter this house, walk through it going through every door exactly once, and then exit again through the other door? If not, could it be done if you are allowed to miss one door, not going through it? [Hint: it may be easier if instead of this picture, you make a new one, replacing each room (and also the outside of the house) by a point, and drawing a line connecting two points for every door.]