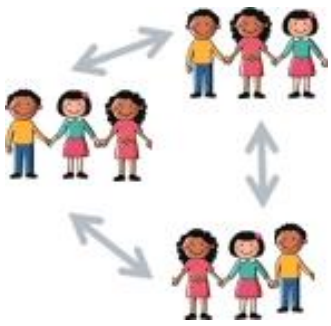
Math 4a. Classwork 19.

In the last class two week ago we were solving permutation problems about number of ways to fix a dinner, or choose a team. Now let us take another look at some of them.

1) How many different 3 digit numbers can we create using 8 digits, 1, 2, 3, 4, 5, 6, 7, and 8 without repetition of the digits, i.e. such numbers that only contain different digits?
2) How many different ways are there to choose a team of 3 students out of 8 to participate in the math Olympiad.

What are the similarities in these two problems? Can you see the difference between them?

In both cases, we have 8 possible ways to choose the first item (digit or student), 7 possible ways to choose the second item, and 6 different ways to choose the third one. So, there are $8 \cdot 7 \cdot 6$ different 3-digit numbers created from digits 1, 2, 3, 4, 5, 6, 7, and 8 and $8 \cdot 7 \cdot 6$ different teams of 3 students out of 8. Or not?



We can create numbers 123, 132, 213, 231, 321, 312 and they are all different numbers. If we chose Mike, Maria, and Jessika, a team of 3 students for the math Olympiad, it doesn't matter in which order we wrote their names.

In the first case, we have $8 \cdot 7 \cdot 6$ ways to create a 3-digit number out of 8 digits. In the second case for each group of 3 kids we will count 6 times (3! – number of ways to put 3 kids in line) more possible choices than there really are.

$$8P(3) = P(8,3) = 8 \cdot (8-1) \cdot (8-2)$$

$$= 8 \cdot (8-3+2) \cdot (8-3+1) - \text{number of way to choose, order matter}$$

$$C(8,3) = \frac{8 \cdot (8-1) \cdot (8-2)}{3!}$$

$$\text{number of ways to choose, order doesn't matter}$$

$$P(3,3) = 3 \cdot 2 \cdot 1 = 3 \cdot (3-3+2) \cdot (3-3+1)$$
$$= 3! - \text{ number of ways to rearrange 3 obgects}$$

In the school cafeteria, there is only limited space to form the line, so only 25 students can be inside simultaneously and form the line. How many ways are there to this line to be formed in the cafeteria, if there are 100 students in school in total?

How many different ways are there to form a Science Olympiad team of 25 students in this school? In which case do you think, order is very important and in which case it is not?

1. There are 10 books on the library shelf. 8 of them are authored by different authors and 2 are from the same author. How many different ways are there to place all these books on a shelf so that 2 books of one author will be next to each other?
2. Mother has 2 apples and 3 pears. Each day she gives one fruit to her kid for lunch. How many different orders are there to give these fruits?

**Let us explore the answer to our two problems a little bit more.

$$P(8,3) = 8 \cdot 7 \cdot 6 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{8!}{5!} = \frac{8!}{(8-3)!}$$

$$0! = 1$$

And

$$C(8,3) = \frac{8 \cdot 7 \cdot 6}{3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{8!}{3! \cdot 5!} = \frac{8!}{3! \cdot (8-3)!}$$

Exercises :

1.Find

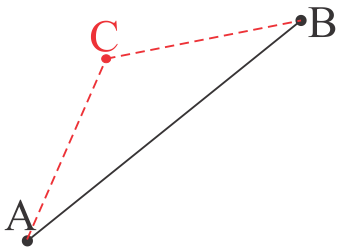
a. 1% from 100
b. 7% from 200
c. 100% from 49
d. 1%from 300
e. 20% from 15
f. 120% from 250
g. 5% from 50
h. 25% from 48
i. 200% from 30

3. Simplify the following expressions:

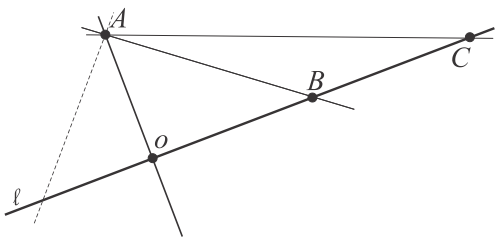
a. $aa^m(-a)^2$;
b. $c^k c(-c^2)c^{k-1}c^3$;
c. $d^n d(-d^{n+1})d^n d^2$;
d. $2x^2y^3 \cdot (-4xy^2)$;
e. $0.5a(-b)^6 \cdot 10a^2b^2$;
f. $\frac{1}{6}(-c)^3 dk \cdot (-6cdk^3)$;
g. $2^4 + 2^4$;
h. $2^m + 2^m$;
i. $2^m \cdot 2^m$;
j. $3^2 + 3^2 + 3^2$;
k. $3^k + 3^k + 3^k$;
l. $3^k \cdot 3^k \cdot 3^k$;

4. **Geometry**.

The shortest distance between two points is a part of a straight line passing through these two points (a segment).



The distance between a point and a line is the distance between the point and the point of intersection of the line and the perpendicular drawn from the point to the line.



AO is a perpendicular drawn from the point A to the line. |AO| is the distance between the point A and the line l.

Distance between two parallel lines is a distance between any point of one line and the other line.

*On a picture on the right the caterpillar wants to go from vertex G to vertex E on the cube. Draw the shortest way for it to go. What will be the shortest way to go from the vertex G to vertex A? Find all possible solutions.