MATH 10
ASSIGNMENT 1: REVIEW OF LAST YEAR
SEPTEMBER 17, 2017

Welcome to the new semester at SchoolNova!!

Here are the main topics we plan to study this year:

- Beginning analysis: properties of real numbers, limits of sequences.
- Coordinate geometry in 2d and 3d. Vectors, dot product; equations of planes and lines. Equations of ellipse, parabola, hyperbola and quadrics in 3d.
- Linear algebra: systems of linear equations, matrices, and more.

Unlike the calculus courses (which many of you take in school or at Stony Brook), we will be more interested in questions "why this works?" rather than "how one uses it to solve an problem from engineering".

We will try to do much of the homework in class so that you do not need to spend too much time on it at home. As usual, all HW assignments and other information will be posted online at `http://www.schoolnova.org` (click on Homework in the navigation bar on top).

I ask that each student bring a notebook (preferably quad ruled), pencils and a folder or binder to keep old assignments — you will need them!

We also plan to participate in two math competitions: Math Kangaroo and American Math Contests (AMC). More details will be given later.

If you have any questions, please contact me by email: `kirillov@schoolnova.org`.

Previous material

I expect that you are familiar with the following:

Complex numbers.

Mathematical induction.

Sets and functions.

Number theory: Modular arithmetic, Chinese remainder theorem, Euler function. .

Today we will review these topics; no new material is given.

Homework

1. (a) Compute $2^{2017} \mod 5$
(b) Compute $2^{2017} \mod 11$
(c) Compute $2^{2017} \mod 55$

2. Find the inverse of 19 mod 24, i.e. solve $19e \equiv 1 \mod 24$.

3. Fibonacci numbers are defined by the rules

$$F_1 = F_2 = 1, \quad F_{n+1} = F_n + F_{n-1} \text{ for } n \geq 2$$

Prove that for any $n \geq 1$, we have

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$$

4. Compute

$$\frac{1}{3+4i}$$

5. Let $z = 1 + i\sqrt{3}$.
(a) Compute z^{2017}.
(b) Compute $1 + z + z^2 + \cdots + z^{29}$.

6. The polynomial $x^3 - 39x + 70$ has three roots. Two of these roots are $x_1 = 2$, $x_2 = 5$. What is the third root?

7. Let $f\colon A \to B$, $g\colon B \to C$ be injective functions. Prove that then the composition $g \circ f\colon A \to C$ is injective.

(Recall that a function $f\colon A \to B$ is injective (one-to-one) if for any $y \in B$, the equation $f(x) = y$ has at most one solution. Equivalently, f is not injective if there exist $x_1, x_2 \in A$ such that $x_1 \neq x_2$, but $f(x_1) = f(x_2)$.)

8. A subset S of the real line is called *bounded* if one can find an interval $[-M, M]$ which cotains all elements of S. Can you write this definition using no words but only quantifiers, variables, arithemtic operations and inequalities?