Accelerated math. Homework 25.
Problems marked with * are more difficult.

1. Simplify the following algebraic fractions, remember, that

$$
\begin{aligned}
& (a \pm b)^{2}=a^{2} \pm 2 a b+b^{2} \\
& (a+b)(a-b)=a^{2}-b^{2}
\end{aligned}
$$

Examples:
a.

$$
\begin{aligned}
& \frac{0.36 x^{2}+1.2 x y+y^{2}}{1.8 x+3 y}=\frac{(0.6 x+y)^{2}}{3(0.6 x+y)}=\frac{(0.6 x+y)(0.6+y)}{3(0.6 x+y)}=\frac{0.6 x+y}{3}=0.2 x+\frac{1}{3} y \\
& 0.6 x+y \neq 0, \\
& \text { b. }
\end{aligned}
$$

$\frac{\frac{1}{4} a-b}{b-\frac{1}{4} a}=\frac{-\left(b-\frac{1}{4} a\right)}{b-\frac{1}{4} a}=-1$
$b-\frac{1}{4} a \neq 0, \quad b \neq \frac{1}{4} a$
c.
$\frac{\frac{1}{9} x^{2}-25}{\frac{2}{3} x+10}=\frac{\left(\frac{1}{3} x+5\right)\left(\frac{1}{3} x-5\right)}{2\left(\frac{1}{3} x+5\right)}=\frac{1}{3} x-5$
$\frac{1}{3} x+5 \neq 0, \quad \frac{1}{3} x \neq-5, \quad x \neq-15$

$$
\begin{array}{ccc}
\frac{5 x-1-3 x}{2-4 x} & \frac{x-1}{x^{2}-1} & \frac{p-p^{2}}{p^{2}-1} \\
\frac{0.64 y^{2}-4.8 y+9}{0.8 y-3} & \frac{n^{2}-m^{2}}{(n-m)^{2}} &
\end{array}
$$

2. Simplify the folloing expressions:

Example:

$$
\begin{aligned}
(x+1)(x+2) & -(x+3)(x+4)=\left(x^{2}+2 x+x+2\right)-\left(x^{2}+4 x+3 x+12\right) \\
& =\left(x^{2}+3 x+2\right)-\left(x^{2}+7 x+12\right)=x^{2}+3 x+2-x^{2}-7 x-12 \\
& =-4 x-10
\end{aligned}
$$

$$
\begin{aligned}
& 2 x+(x-1)(x+1) ; \\
& 7 p^{2}-(p+1)(p+2) ; \\
& (a+2)(a-1)-(a+1)(a-2) ; \\
& (p+2)(p-1)+(p+3)(p-5) ; \\
& (4-x)(2-x)-(x+2)(1-x) .
\end{aligned}
$$

3. May the sum of
a. 2 odd numbers
b. 2 even numbers
c. even and odd numbers be a prime number? Explain why.
4. 

a. Prove that among any three consecutive odd numbers one will be divisible by 3.
b. Numbers $p, p+2, p+4$ are prime numbers. Find $p$, prove that $p$ is unique, so there is no any other $p$, such that satisfy this condition.
5. Prove the following identities:

$$
\begin{aligned}
& (a+b)^{2}=a^{2}+2 a b+b^{2} \\
& (a-b)^{2}=a^{2}-2 a b+b^{2} \\
& (a+b)(a-b)=a^{2}-b^{2}
\end{aligned}
$$

Using the similar approach find the expression for
$(a+b)^{3}=$
$(a-b)^{3}=$
6. Draw a circle (use compass). On the circle mark three arbitrary points. Draw a triangle using the marked points as vertices of the triangle (use ruler). Find midpoints of all three sides of the triangle. Draw perpendiculars to the sides of the triangle through their midpoints. Did they intersect at the center of the circle? Should they intersect at the center of the circle? Why?

