Accelerated Math. Class work 17.

**Geometry**.

Rectangle is a quadrilateral with all 4 right angles. Based on this definition the equality of opposite sides can be proven, as well as the fact that opposite sides are parallel, so a rectangle is a parallelogram. A square is a rectangle with all 4 equal sides. What is rhombus? Is a square a rhombus? Draw the Venn diagrams of the following sets:

A – is a set of quadrilaterals,

B – is a set of parallelograms

C – is a set of squares, and

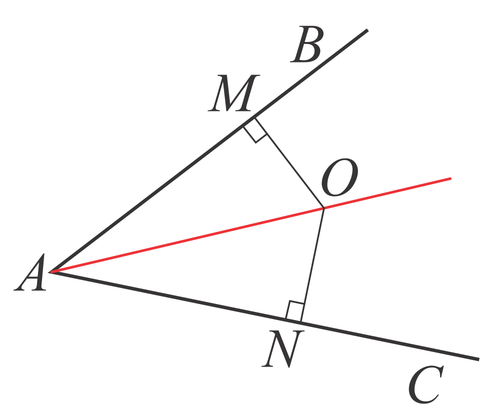
D – is a set of rhombuses.

**Distance between the point and the line.**

The **distance from a point to a line** is the length of the line segment which joins the point to the line and is perpendicular to the line.

**Theorem**. All points of a bisector of an angle are located at an equal distance from both sides of the angle.

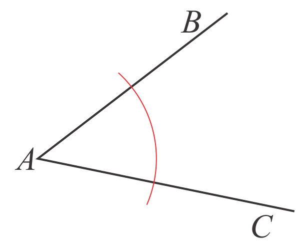
Ray $\overrightarrow{AO}$ is a bisector of the angle $\angle BAC$. Segment $[OM]\perp\overrightarrow{AB}$ and $[ON]\perp\overrightarrow{AC}$. We need to prove that $|OM| = |ON|$.



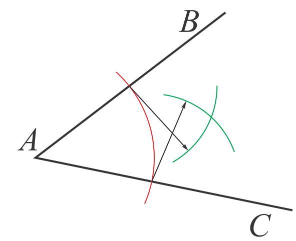

Angle $\angle MAO$ is equal to the angle $\angle OAN$, since the AO is bisector. Angles $\angle AMO$ and $\angle ANO$ are both right angles, therefore angles $\angle MOA$ and $\angle NOA$ are also equal and the segment $[AO]$ is the common side, two triangles $\Delta AMO$ and $\Delta ANO$ are congruent by ASA criteria and $|OM| = |ON|$.

Construction problem:

Draw the bisector of an angle.

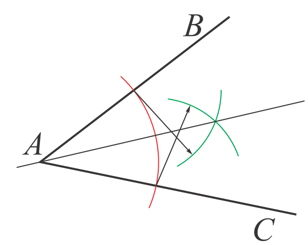



1. Bisector is a median and an altitude in an isosceles triangle.
2. To construct a bisector mark two points on both sides of the angle on the same distance from the vertex of the angle.




3. With the same radius (greater than half of the length between two marked points) draw two arches with centers at marked points.

4. Draw a line through the vertex of the angle and the point of intersection of two arches.




Explain each step.

Exercises.

1. Construct the angle congruent to a given angle.
2. The sum of the lengths of the sides AB and BC of a triangle equals to the segment

The sum of the lengths of the sides BC and CA equals to the segment

And the sum of the lengths of the sides CA and AB equals to the segment

Construct the triangle ABC.

**Algebra**.

Factoring.

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a + b)(a + b) = aa + ab + ba + bb = a^2 + 2ab + b^2$

Let's see how we can factorize the expression $a^2 + 2ab + b^2$ without knowing that it is actually $(a + b)^2$.

$a^2 + 2ab + b^2 = a^2 + b^2 + ab + ab = a^2 + ab + b^2 + ab = a(a + b) + b(a + b)$
$= (a + b)(a + b)$

Second very useful identity is the difference of two sqares:

$(a-b)(a+b) = a^2 + ab - ba - b^2 = a^2 - b^2$

Can the expression $a^2 - b^2$ be factorize without knowing the left part?

$$a^2 - b^2 = a^2 - b^2 + ab - ab = a^2 + ab - b^2 - ab = a(a+b) - b(a+b)$$
$$= (a+b)(a-b)$$

Factorize the following expressions:

$x^2 - 3x + 2;$

$a^2 - 6a + 5;$

$m^2 - 3mn + 2n^2;$

$ab + cb + ad + cd;$

$a^2 - 2ab + b^2 - c^2$