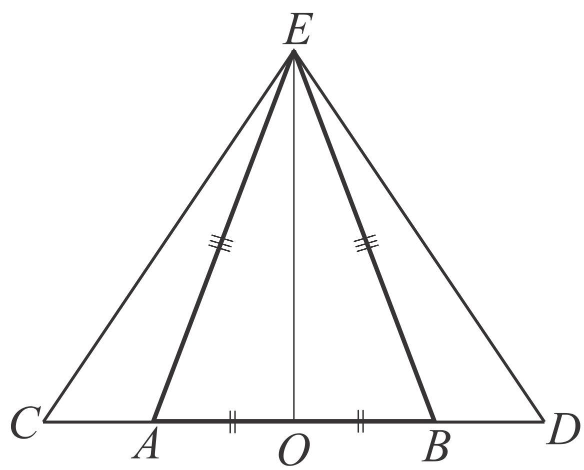
Accelerated Math. Class work 14.

Geometry.

1. Two segments, AB and CD are located on the same line in a way that point O is a midpoint for both segments and point E lies outside of the line. Prove that if the triangle ABE is an isosceles triangle with |AE|=|BE|, the triangle CED is an isosceles tringle as well.



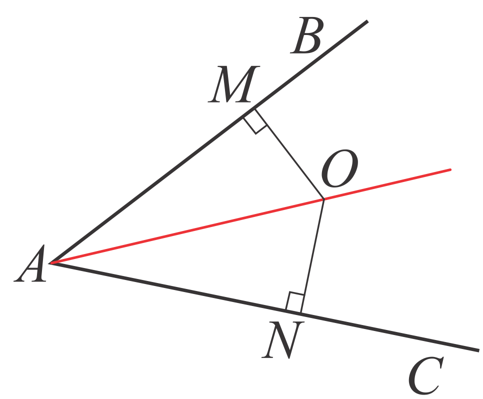
Distance between the point and the line.

The **distance from a point to a line** is the length of the line segment which joins the point to the line and is perpendicular to the line.

Theorem. All points of a bisector of an angle are located at an equal distance from both sides of the angle.

Ray $\overrightarrow{AO}$ is a bisector of the angle $\angle BAC$. Segment $[OM]\perp\overrightarrow{AB}$ and $[ON]\perp\overrightarrow{AC}$. We need to prove that $|OM| = |ON|$.

Angle $\angle MAO$ is equal to the angle $\angle OAN$, since the AO is bisector. Angles $\angle AMO$ and $\angle ANO$ are both right angles, therefore angles $\angle MOA$ and $\angle NOA$ are also equal and the segment $[AO]$ is the common side, two triangles ΔAMO and ΔANO are congruent by ASA criteria and $|OM| = |ON|$.



Algebra.

We already know what is GCD and LCM for several natural numbers and we know how to find them.

Exercise:

Find GCD (GCF) and LCM for numbers

a. 222 and 345.
b. $2^2 \cdot 3^3 \cdot 5$ and $2 \cdot 3^2 \cdot 5^2$

Can we apply the same strategy to find CF and CM for algebraic expressions? (In this case the concept of GCD and LCM cannot be applied.) For example, can CF and CM be found for expressions $2x^2y^5$ and $4x^3y^2$? x and y are variables and can't be represented as a product of factors, but they itself are factors, and the expression can be represented as a product:

$2x^2y^5 = 2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y,$ $\qquad$ $4x^3y^3 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot$

$A = (Factors, 2x^2y^5) = \{2, x, x, y, y, y, y, y\}, B = (Factors, 4x^3y^3) = \{ 2, 2, x, x, x, y, y\}$

Common devisors are any product of $A \cap B = \{ 2, x, x, y, y\}$.
What about common multiples? Product of all factors of both numbers (or the product of two numbers) will be the multiple, but minimal common multiple will be the product of the

$$A \cup B = \{2,2, x, x, x, y, y, y, y, y\}$$

$$\frac{2x^2y^5}{2 \cdot x^2y^2} = y^3; \qquad \frac{4x^3y^2}{2 \cdot x^2y^2} = 2x;$$

$$\frac{2x^3y^5}{2 \cdot x^2y^5} = x; \qquad \frac{4x^3y^5}{4 \cdot x^3y^3} = y^2;$$

Find common factors and common multiples of: