Accelerated math. Homework 11.

Problems marked with \* are more difficult.



1. Compute:

a. 
$$(-3)^3$$
;f.  $(-2)^7$ ;j.  $\frac{1}{3^2}$ ;b.  $-3^3$ ;g.  $(2 \cdot 3)^3$ ;k.  $3^{-2}$ ;c.  $(-3)^4$ ;h.  $2 \cdot 3^3$ ;l.  $(-3)^{-2}$ ;d.  $-3^3$ ;i.  $\left(\frac{1}{3}\right)^2$ ;m.  $(-5 \cdot 2)^3$ 

a. 
$$(-3)^3 = (-3) \cdot (-3) \cdot (-3) = -27;$$
  
b.  $-3^3 = -(3 \cdot 3 \cdot 3) = -27;$   
c.  $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81;$   
d.  $-3^3 = -(3 \cdot 3 \cdot 3) = -27; -3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$   
e.  $-2^7 = -(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = -128;$   
f.  $(-2)^7 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -128;$   
g.  $(2 \cdot 3)^3 = 6^3 = 6 \cdot 6 \cdot 6 = 216 = 2^3 \cdot 3^3 = 8 \cdot 27 = 216;$   
h.  $2 \cdot 3^3 = 2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 27 = 54;$   
i.  $\left(\frac{1}{3}\right)^2 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3\cdot 3} = \frac{1}{9};$   
j.  $\frac{1}{3^2} = \frac{1}{3\cdot 3} = \frac{1}{9};$   
k.  $3^{-2} = \frac{1}{3^2} = \frac{1}{9};$   
l.  $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9};$   
m.  $(-5 \cdot 2)^3 = (-10)^3 = -1000$ 

Remember, that  $a^n : a^m = a^{n-m} = a^{n+(-m)} = a^n \cdot \frac{1}{a^m} = a^n \cdot a^{-m}$ 

2. Prove that values of the following expressions do not depend from the value of variables. Find these values. Hint: simplify these expressions.

a) 
$$\frac{4^m + 4^m + 4^m + 4^m}{4^m : 4^2}$$
; b)  $\frac{10^{pa3}}{10^n + 10^n + \dots + 10^n}$ ; c)  $\frac{99^{pa3}}{99^{k} + 99^k + \dots + 99^k}{99^{k+2} : 99}$ .  
a.  $\frac{4^m + 4^m + 4^m + 4^m}{4^m : 4^2} = \frac{4 \cdot 4^m}{4^m : \frac{1}{4^2}} = 4 : \frac{1}{4^2} = 4 \cdot 4^2 = 4^3 = 64$   
b.  $\frac{10 \text{ times}}{10^n + 10^n + \dots + 10^n} = \frac{10 \cdot 10^n}{10^n : 10} = \frac{10 \cdot 40^n}{4^{10} \cdot \frac{1}{10}} = 10 : \frac{1}{10} = 10 \cdot 10 = 100$ 

c. 
$$\overline{\frac{99 \text{ times}}{99^{k}+99^{k}+\dots+99^{k}}}_{99^{k}:99} = \frac{99 \cdot 99^{k}}{99^{k} \cdot \frac{1}{99}} = 99 \cdot \frac{1}{99} = 99 \cdot 99 = 9801$$

3. a. Prove, that in isosceles triangle medians conducted to the equal sides are equal. Triangle ABC is an isosceles triangle. [CM] and [AP] are medians at the base. Prove that |CM| = |AP|.

 $\triangle AMC \cong \triangle APC$  by SAS criteria. (Side AC is a common side, |AM| = |CP| as the halves of the sides AB and CB,  $\angle MAC \cong \angle PCA$  as two angles at the base). So, |CM| = |AP|.

b. \*\*(This problem is much more difficult than the problem a. Just write any idea you can come up with). Prove, that if two medians in a triangle are equal, this triangle is an isosceles triangle.

Be careful about what you know and what you want to prove in each case. Hint: look for congruent (equal) triangles.

What about altitudes? Can you formulate similar statement about altitudes? In an isosceles triangle altitudes, drawn to the equal sides are equal. The prove is very similar:

 $\triangle AMC \cong \triangle APC$  by ASA criteria. (In both tringles there is one right angle,  $\angle MAC \cong \angle PCA$  as two angles at the base, so  $\angle MCA \cong \angle PAC$ , side AC is a common side.)

Hint: look for congruent (equal) triangles.

- 4. Draw a triangle with sides 5 cm, 7 cm, and 7 cm (use ruler and compass). Mark the midpoints of equal sides (use ruler), draw medians to the equal sides. Measure these two medians. Are they equal?
- 5. Draw the triangle with sides 6 cm, 8 cm, and 8 cm (use ruler and compass). Draw all two altitudes (use ruler and anything which has a right angle) to the equal side. Measure them. Are they equal?

