Accelerated Math. Class work 13.

Algebra.

1. Rewrite the following algebraic fractions (expressions) as a single fraction:

a. $\frac{1}{2}+\frac{2}{x}$;

b. $\frac{1}{x}+\frac{1}{y}$;

c. $\frac{2a}{b}-\frac{2}{b^2}$;

d. $\frac{2}{3}\cdot\frac{a}{b}$;

2. Rewrite without parenthesis:

a. $a+(b-c+d)$;

b. $a-(b-c-d)$;

c. $a-(b+c+d)$;

d. $a+(b+c-d)$;

e. $(a-b)+(c-d)$;

f. $(x+y)-(z+t)$;

g. $(a-b)-(c-d)$;

h. $(a+b)+(-c-d)$;

3. Rewrite without parenthesis and simplify the following expressions.

а) $(x+y)+(y-x)$;

б) $(a-b)-(a-b)$;

в) $(c-d)-(c+d)$;

г) $(u+v)-(v-u)$;

д) $m-(n-p-m)$;

е) $(a+b)-(b+c)-(a-c)$;

ж) $(k+m)-(k-m)+(m-k)$;

з) $(b+1)-(a-1)-(b-a)$.

4. What should be inside the parenthesis to get the final expression:

а) $x-(\ldots)=x-a+b-c$; б) $x-y=(x-a)+(\ldots)$.

5. Prove that a natural number written with 3 same digits is divisible by 37.
6. Prove that a natural number written with 4 same digits is divisible by 11 and 101.
7. Compute:

a. $\frac{5^n+5^n+5^n+5^n+5^n}{5^n+5^n+5^n+5^n}$;

b. $\frac{\overbrace{100^n+100^n+\ldots+100^n}^{100\ times}}{\underbrace{100^n+100^n+\cdots+100^n}_{90\ times}}$

8. Simplify the following fractions:

a. $\frac{2m+3m}{5m}$;

b. $\frac{2ab-2a}{4bc-4c}$;

c. $\frac{a^2-a}{a^3-a^2}$;

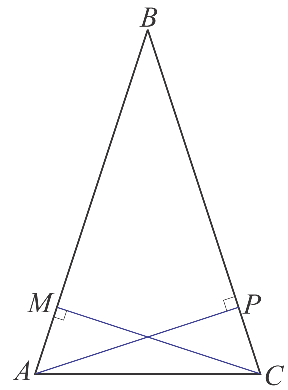
d. $\frac{(m-c)^2}{c^3-cm}$;

e. $\frac{m^2-2mn+n^2}{an-am}$;

f. $\frac{ax+ay-bx-by}{x^2+xy}$

Geometry.

Theorem (direct theorem).
In isosceles triangle two altitudes, drawn to the equal sides are equal.



Let ABC be an isosceles triangle with altitudes CM and AP. In other words, $|AB| = |BC|$ and we need to prove that $|AP| = |CM|$
$\Delta AMC \cong \Delta APC$ (Triangles AMC and APC are congruent) by ASA criteria. In both tringles there is one right angle, $\angle MAC \cong \angle PCA$ as two angles at the base of an isosceles triangle, therefore angles $\angle MCA$ and $\angle PAC$ are also equal, side AC is a common side, so $\angle MCA \cong \angle PAC$, therefore $|AP| = |CM|$.

Let's prove the converse theorem. The converse theorem is a theorem obtained by reversing role of premise and conclusion of the initial (direct) theorem. Premise gives us the information needed to establish that the statement (conclusion) is true. In the direct theorem about altitudes in the isosceles triangle, the premise is the information that the given triangle is an isosceles triangle and two altitudes are drawn to the equal sides. The conclusion is the statement that they are equal.

To formulate the converse theorem the conclusion and the premise should reverse their roles. Conclusion should become the given information:

If in a triangle (general triangle) two altitudes are equal

And the premise should be converted into the conclusion (the statement which need to be proven):

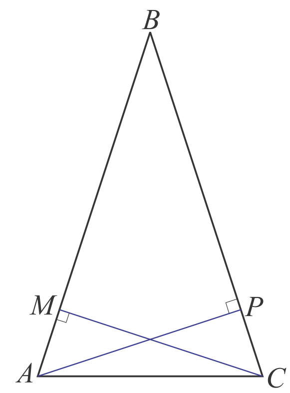
this triangle is an isosceles triangle (sides to which altitudes are drown are equal).

To prove that the triangle is an isosceles triangle we have either to prove directly that two sides are equal or that two angles are equal (we already know that in the isosceles triangle angles at the base are equal).

Let again in the triangle ABC altitude |CM| to be equal to the altitude |AP|. We need to prove the ABC is isosceles triangle.

Proof 1:

The area of the triangle is



$$S = \frac{1}{2}|CM| \cdot |AB| = \frac{1}{2}|AP| \cdot |CB| = \frac{1}{2}h \cdot |AB| = \frac{1}{2}h \cdot |BC|$$

Therefore |AB|=|BC|.

Proof 2:

$\Delta BMC \cong \Delta BPA$,

because angle $\angle ABP$ and $\angle MBC$ is a common angle, $\angle CMP = \angle APB = 90°$, therefore $\angle BAP = \angle BCM$, $|AP| = |CM|$.

So, $|AB| = |BC|$.

Formulate similar direct theorem about bisectors in the isosceles triangle. Prove it. Formulate the converse theorem.