Problems marked with * are more difficult.

1. To solve the following problem, write an equation and solve it:
a. Measures of three angles of a triangle in degrees, are represented by three consecutive natural numbers. Find these angles.

Three consecutive numbers can be represented as
$x, x+1, x+2$. The sum of the three angles of a triangle is $180^{\circ}$, therefor
$x+x+1+x+2=180$
$3 x+3=180$
$3 x=180-3=177$
$x=177 \div 3=59$
Three angles are 59, 60, and 61.
(This problem can be solved without solving the equation. In the equilateral triangle all three angles ae $60^{\circ}$, so three consecutive numbers will be 59,60 , and 61 ).
b. Measures of three angles of a triangle in degrees, are represented by three consecutive even natural numbers. Find these angles.

Three consecutive even (or odd) numbers can be represented as
$x, x+2, x+4$. The sum of the three angles of a triangle is $180^{\circ}$, therefor
$x+x+2+x+4=180$
$3 x+6=180$
$3 x=180-6=174$
$x=174 \div 3=58$
Three angles are 58, 60, and 62.
c. Measures of three angles of a triangle in degrees, are represented by three consecutive multiples of 3 . Find these angles.

Three consecutive even multiples of 3 can be represented as
$x, x+3, x+6$. The sum of the three angles of a triangle is $180^{\circ}$, therefor
$x+x+3+x+6=180$
$3 x+9=180$
$3 x=180-9=171$
$x=174 \div 3=57$
Three angles are 57, 60, and 63.
2. Draw a triangle with sides $5 \mathrm{~cm}, 6 \mathrm{~cm}$, and 7 cm (use ruler and compass). Mark the midpoint of each side (use ruler), draw all three medians.

3. Draw the triangle with sides $5 \mathrm{~cm}, 6 \mathrm{~cm}$, and 9 cm (use ruler and compass). Draw all three altitudes (use ruler and anything which has a right angle).

4. Open parenthesis, combine like terms and simplify the expression:

$$
\begin{aligned}
c-(c-d)- & \left(c-\frac{d}{2}\right)-\left(c-\frac{d}{4}\right)-\left(c-\frac{d}{8}\right)-\left(c-\frac{d}{16}\right)+\frac{d}{16} \\
& =c-c+d-c+\frac{d}{2}-c+\frac{d}{4}-c+\frac{d}{8}-c+\frac{d}{16}+\frac{d}{16}=-4 c+2 d
\end{aligned}
$$

5. Two gears are in in clutch. One gear has 18 cogs, and another has 63 . How many turns will each gear make before they both return to their original position?
The original position is marked with yellow dots on both gears. We have to find how many full terns should be passed by each gear to come back to the starting position. The Least Common Multiple (LCM) of 18 and 63 is $18=2 \cdot 3 \cdot 3, \quad 63=3 \cdot 3 \cdot 7, \quad L C M=2 \cdot 3 \cdot 3 \cdot 7=126$

$126 \div 18=7, \quad 126 \div 63=2$
Answer; small gear with 18 cogs will do 7 full turns and the big one with 63 cogs will turn only twice before they return to the original position for the first time.
6. Theorem. In isosceles triangle the bisector passed to the base (in isosceles triangle the base is the side different from two equal sides) is a median and an altitude as well.


Let the triangle $\triangle A B C$ be an isosceles triangle, such that $A B=B C$, and $B M$ is a bisector. We need to prove that $B M$ is a median and an altitude, which means that we need to prove that $A M=M C$ and angle $\angle B M C$ is a right angle.
$B M$ is a bisector, so $\angle A B M$ and $\angle M B C$ are equal (congruent) engles, the triangle $\triangle A B C$ is an isosceles triangle, so $A B=B C$ and the segment $M B$ is common side for triangles $\triangle A B M$ and $\triangle M B C$. Based on the Side-Angle-Side criteria, the triangles $\triangle A B M$ and $\triangle M B C$ are congruent. Therefore, $A M=M C$ and $B M$ is a median. Also we can now see that angles $\angle A$ and $\angle C$ are congruent. (Isosceles triangle has equal angles adjacent to the base).
$\angle A+\angle B+\angle C=180^{\circ}=2 \angle A+\angle B \Rightarrow 90^{\circ}=\angle A+\frac{1}{2} \angle B$
but for the triangle ABM (as well as for MBC), $\angle A+\frac{1}{2} \angle B+\angle B M A=180^{\circ}$, therefore $\angle B M A=90^{\circ}$ and BM is also an altitude.

Please, be ready to prove this theorem.
7. Is the number $49^{4} \cdot 6^{2}$ divisible by 7 ? By 14 ? By 42 ?
$49^{4} \cdot 6^{2}=(7 \cdot 7)^{4} \cdot(2 \cdot 3)^{2}=7^{4} \cdot 7^{4} \cdot 2^{2} \cdot 3^{2}=7 \cdot 7^{3} \cdot 2^{2} \cdot 3^{2}=14 \cdot 7^{3} \cdot 2^{1} \cdot 3^{2}=42 \cdot 7^{3} \cdot 2^{1} \cdot 3^{1}$. Since $14=7 \cdot 2$,
$42=6 \cdot 7$
$49^{4} \cdot 6^{2}$ is divisible by 7,14 , an 42 .
8. Simplify the following expressions:
a. $a a^{m}(-a)^{2}$;
b. $c^{k} c\left(-c^{2}\right) c^{k-1} c^{3}$;
c. $d^{n} d\left(-d^{n+1}\right) d^{n} d^{2}$;
d. $2 x^{2} y^{3} \cdot\left(-4 x y^{2}\right)$;
e. $0.5 a(-b)^{6} \cdot 10 a^{2} b^{2}$;
f. $\frac{1}{6}(-c)^{3} d k \cdot\left(-6 c d k^{3}\right)$;
g. $2^{4}+2^{4}$;
h. $2^{m}+2^{m}$;
i. $2^{m} \cdot 2^{m}$;
j. $3^{2}+3^{2}+3^{2}$;
k. $3^{k}+3^{k}+3^{k}$;

1. $3^{k} \cdot 3^{k} \cdot 3^{k}$;
a. $\quad a a^{m}(-a)^{2}=a a^{m} a^{2}=a^{m+3}$
b. $c^{k} c\left(-c^{2}\right) c^{k-1} c^{3}=-c^{k} c^{1} c^{2} c^{k-1} c^{3}=-c^{k+1+2+k-1+3}=-c^{2 k+5}$
c. $d^{n} d\left(-d^{n+1}\right) d^{n} d^{2}=-d^{n} d d^{n+1} d^{n} d^{2}=-d^{n+1+n+1+n+2}=-d^{3 n+4}$
d. $2 x^{2} y^{3} \cdot\left(-4 x y^{2}\right)=-8 x^{2} y^{3} x y^{2}=-8 x^{2+1} y^{3+2}=-8 x^{3} y^{5}$
e. $0.5 a(-b)^{6} \cdot 10 a^{2} b^{2}=5 a^{1} b^{6} a^{2} b^{2}=5 a^{3} b^{8}$
f. $\frac{1}{6}(-c)^{3} d k \cdot\left(-6 c d k^{3}\right)=\frac{1}{6} \cdot(-6) \cdot\left(-c^{3}\right) d k c d k^{3}=c^{4} d^{2} k^{4}$
g. $2^{4}+2^{4}=2 \cdot 2^{4}=2^{5}$
h. $2^{m}+2^{m}=2 \cdot 2^{m}=2^{m+1}$
i. $\quad 2^{m} \cdot 2^{m}=2^{m+m}=2^{2 m}$
j. $3^{2}+3^{2}+3^{2}=3 \cdot 3^{2}=3^{3}$
k. $3^{k}+3^{k}+3^{k}=3 \cdot 3^{k}=3^{k+1}$
2. $3^{k} \cdot 3^{k} \cdot 3^{k}=3^{k+k+k}=3^{3 k}=\left(3^{k}\right)^{3}=\left(3^{3}\right)^{k}$

Remember, that $(-a)^{k} \neq\left(-a^{k}\right)$, in the first case $(-a)^{k}=\underbrace{(-a) \cdot(-a) \cdot \ldots \cdot(-a)}_{k \text { times }}$, but in the second one $\left(-a^{k}\right)=-\underbrace{a \cdot a \cdot \ldots \cdot a}_{k \text { times }}$

## Example:

$$
\begin{gathered}
\frac{1}{6}(-c)^{3} d k \cdot\left(-6 c d k^{3}\right)=\frac{1}{6} \cdot(-6)(-c)^{3} c d d k k^{3}=-1 \cdot((-c) \cdot(-c) \cdot(-c)) c d d k k^{3} \\
=-1 \cdot\left(-c^{3}\right) c d d k k^{3}=-1 \cdot\left(-c^{4}\right) d^{2} k k^{3}=c^{4} d^{2} k^{4}
\end{gathered}
$$

9. Compare (replace $\ldots$ with $>,<$, or $=$ ) if possible, if it is known that $a$ and $b$ are positive numbers and $x$ and $y$ are negative numbers:

| $0 \ldots x$ | a ... 0 | -b ... 0 | $0 \ldots-x$ |
| :---: | :---: | :---: | :---: |
| $a \ldots x$ | $y \ldots b$ | $-y \ldots x$ | $-a . . . b$ |
| $\|x\| \ldots x$ | $-\|y\| \ldots y$ | $a$... $\|a\|$ | $\|b\| \ldots \mid-b$ |
| $\|x\| \ldots a$ | $\|x\| \ldots-x$ | $\|x\| \ldots-\|y\|$ | $a \ldots\|-b\|$ |
| $0>x$ | $a>0$ | $-b>0$ | $0<-x$ |
| $a>x$ | $y<b$ | $-y>x$ | $-a<b$ |
| $\|x\|>x$ | $-\|y\|=y$ | $a=\|a\|$ | $\|b\|=\|-b\|$ |
| $\|x\| \ldots a$ | $\|x\|=-x$ | $\|x\|>-\|y\|$ | $a \quad . . .\|-b\|$ |

10. Positive or negative value of $m$ will make the following equalities true statements?

$$
\begin{aligned}
& |m|=m, \text { if } m \geq 0 \\
& |m|=-m, \text { if } m \leq 0 \\
& -m=|-m|, \text { if } m \leq 0 \\
& m=|-m|, \text { if } m \geq 0
\end{aligned}
$$

$$
m=-m, \text { if } m=0
$$

$$
m+|m|=0, \text { if } m \leq 0
$$

$$
m+|m|=2 m \text { if } m \geq 0
$$

$$
m-|m|=2 m, \text { if } m \leq 0
$$

